Demystifying Electric Grid Application of Measurement-Based Modal Analysis

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Overview

• Electric grids are in a time of rapid transition, with lots of positive developments. It is a very exciting time to be in the field! However, there are also lots of challenges.

• To meet these challenges we need to widely leverage tools from other domains and make them useful

• This webinar presents one such tool, the application of measurement-based modal analysis techniques for large-scale electric grids
Signals

Throughout the talk I’ll be using the term “signal,” which has several definitions.

A definition from Merrian-Webster is:
- “A detectable physical quantity or impulse by which messages or information can be transmitted.”

A common electrical engineering definition is “any time-varying quantity.”

Our focus today is on such time-varying signals, particularly associated with oscillations.
Oscillations

• An oscillation is just a repetitive motion that can be either undamped, positively damped (decaying with time) or negatively damped (growing with time).

• If the oscillation can be written as a sinusoid then

\[ e^{\alpha t} \left( a \cos(\omega t) + b \sin(\omega t) \right) = e^{\alpha t} C \cos(\omega t + \theta) \]

where \( C = \sqrt{A^2 + B^2} \) and \( \theta = \tan\left(\frac{-b}{a}\right) \)

• The damping ratio is

\[ \xi = \frac{-\alpha}{\sqrt{\alpha^2 + \omega^2}} \]

The percent damping is just the damping ratio multiplied by 100; goal is sufficiently positive damping.
Power System Oscillations

• Power systems can experience a wide range of oscillations, ranging from highly damped and high frequency switching transients to sustained low frequency (< 2 Hz) inter-area oscillations affecting an entire interconnect

• Types of oscillations include
  – Transients: Usually high frequency and highly damped
  – Local plant: Usually from 1 to 5 Hz
  – Inter-area oscillations: From 0.15 to 1 Hz
  – Slower dynamics: Such as AGC, less than 0.15 Hz
  – Subsynchronous resonance: 10 to 50 Hz (less than synchronous)
Example Oscillations

- The left graph shows an oscillation that was observed during a 1996 WECC Blackout, the right from the 8/14/2003 blackout.
Small Signal Analysis and Measurement-Based Modal Analysis

• Small signal analysis has been used for decades to determine power system frequency response
  – It is a model-based approach that considers the properties of a power system, linearized about an operating point

• Measurement-based modal analysis determines the observed dynamic properties of a system
  – Input can either be measurements from devices (such as PMUs) or dynamic simulation results
  – The same approach can be used regardless of the measurement source

• Focus here is on the measurement-based approach
Ring-down Modal Analysis

• Ring-down analysis seeks to determine the frequency and damping of key power system modes following some disturbance

• There are several different techniques, with the Prony approach the oldest (from 1795)

• Regardless of technique, the goal is to represent the response of a sampled signal as a set of exponentially damped sinusoidals (modes)

\[ y(t) = \sum_{i=1}^{q} A_i e^{\sigma_i t} \cos(\omega_i t + \phi_i) \]

\[
\text{Damping (\%)} = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_i^2}} \times 100
\]
Where We Are Going: Extracting the Modes from Signals

• The goal is to gain information about the electric grid by extracting modal information from its signals
  – The frequency and damping of the modes is key

• The premise is we’ll be able to reproduce a complex signal, over a period of time, as a set of sinusoidal modes
  – We’ll also allow for linear detrending

\[ 0.1t + \cos(2\pi 2t) \]
Example: The Summation of two damped exponentials

- This example was created by going from the modes to a signal
- We’ll be going in the opposite direction (i.e., from a measured signal to the modes)
Some Reasonable Expectations

• **Verifiable** to show how well the modes match the original signal(s)
  – We’ll show this

• **Flexible** to handle between one and many signals
  – We’ll go up to simultaneously considering 40,000 signals

• **Fast**
  – What is presented will be, with a discussion of the computational scaling

• **Easy to use**
  – This is software implementation specific; results shown here were done using the modal analysis tool integrated into PowerWorld Simulator (version 22)
Example: One Signal

This could be any signal; image shows the result of the original signal (blue) and the reproduced signal (red)
Verification: Linear Trend Line Only
Verification:
Linear Trend Line + One Mode
Verification: Linear Trend Line + Two Modes
Verification: Linear Trend Line + Three Modes
Verification: Linear Trend Line + Four Modes
Verification: Linear Trend Line + Five Modes

It is hard to tell a difference on this one, illustrating that modes manifest differently in different signals.
A Larger Example We’ll Finish With

Applying the developed techniques to the response of all 43,400 substation frequencies from an 110,000 bus electric grid (20 million plus values)
Measurement-Based Modal Analysis

• There are a number of different approaches
• The idea of all techniques is to approximate a signal, $y_{org}(t)$, by the sum of other, simpler signals (basis functions)
  – Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
  – Properties of the original signal can be quantified from basis function properties
    • Examples are frequency and damping
  – Signal is considered over time with $t=0$ as the start
• Approaches sample the original signal $y_{org}(t)$
Measurement-Based Modal Analysis

• Vector \( y \) consists of \( m \) uniformly sampled points from \( y_{\text{org}}(t) \) at a sampling value of \( \Delta T \), starting with \( t=0 \), with values \( y_j \) for \( j=1 \ldots m \)
  
  – Times are then \( t_j = (j-1)\Delta T \)
  
  – At each time point \( j \), the approximation of \( y_j \) is

\[
\hat{y}_j(t_j, \alpha) = \sum_{i=1}^{n} b_i \phi_i(t_j, \alpha)
\]

where \( \alpha \) is a vector with the real and imaginary eigenvalue components, with \( \phi_i(t_j, \alpha) = e^{\alpha_i t_j} \) for \( \alpha_i \) corresponding to a real eigenvalue, and

\[
\phi_i(t_j, \alpha) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j) \quad \text{and} \quad \phi_{i+1}(\alpha) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)
\]

for a complex eigenvector value
Measurement-Based Modal Analysis

• Error (residual) value at each point $j$ is

$$r_j(t_j, \alpha) = y_j - \hat{y}_j(t_j, \alpha)$$

• The closeness of the fit can be quantified using the Euclidean norm of the residuals

$$\frac{1}{2} \sum_{j=1}^{m} (y_j - \hat{y}_j(t_j, \alpha))^2 = \frac{1}{2} \| r(\alpha) \|_2^2$$

• Hence we need to determine $\alpha$ and $b$

$$\hat{y}_j(t_j, \alpha) = \sum_{i=1}^{n} b_i \phi_i(t_j, \alpha)$$
Sampling Rate and Aliasing

• The Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency
  – For example, to see a 5 Hz frequency we need to sample the signal at a rate of at least 10 Hz

• Sampling shifts the frequency spectrum by $1/T$ (where $T$ is the sample time), which causes frequency overlap.

• This is known as aliasing, which can cause a high frequency signal to appear to be a lower frequency signal
  – Aliasing can be reduced by fast sampling and/or low pass filters.

One Solution Approach: The Matrix Pencil Method

• There are several algorithms for finding the modes. We’ll use the Matrix Pencil Method
  – This is a newer technique for determining modes from noisy signals (from about 1990, introduced to power system problems in 2005); it is an alternative to the Prony Method (which dates back to 1795, introduced into power in 1990 by Hauer, Demeure and Scharf)

• Given m samples, with L=m/2, the first step is to form the Hankel Matrix, $\mathbf{Y}$ such that

\[
\mathbf{Y} = \begin{bmatrix}
y_1 & y_2 & \cdots & y_{L+1} \\
y_2 & y_3 & \cdots & y_{L+2} \\
\vdots & \vdots & \ddots & \vdots \\
y_{m-L} & y_{m-L+1} & \cdots & y_m
\end{bmatrix}
\]

This not a sparse matrix

Algorithm Details, cont.

• Then calculate $\mathbf{Y}$’s singular values using an economy singular value decomposition (SVD)
  \[ \mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \]

• The ratio of each singular value is then compared to the largest singular value $\sigma_c$; retain the ones with a ratio $>$ than a threshold
  – This determines the modal order, $M$
  – Assuming $\mathbf{V}$ is ordered by singular values (highest to lowest), let $\mathbf{V}_p$ be the matrix with the first $M$ columns of $\mathbf{V}$

The computational complexity increases with the cube of the number of measurements!

This threshold is a value that can be changed; decrease it to get more modes.
Aside: The Matrix Singular Value Decomposition (SVD)

• The SVD is a factorization of a matrix that generalizes the eigendecomposition to any m by n matrix to produce

$$Y = U\Sigma V^T$$

where $\Sigma$ is a diagonal matrix of the singular values

• The singular values are non-negative, real numbers that can be used to indicate the major components of a matrix (the gist is they provide a way to decrease the rank of a matrix)

The original concept is more than 100 years old, but has found lots of recent applications
Aside: SVD Image Compression Example

Images can be represented with matrices. When an SVD is applied and only the largest singular values are retained the image is compressed.

Image Source: www.math.utah.edu/~goller/F15_M2270/BradyMathews_SVDImage.pdf
Matrix Pencil Algorithm Details, cont.

• Then form the matrices $V_1$ and $V_2$ such that
  – $V_1$ is the matrix consisting of all but the last row of $V_p$
  – $V_2$ is the matrix consisting of all but the first row of $V_p$

• Discrete-time poles are found as the generalized eigenvalues of the pair $(V_2^TV_1, V_1^TV_1) = (A,B)$

• These eigenvalues are the discrete-time poles, $z_i$ with the modal eigenvalues then

$$\lambda_i = \frac{\ln(z_i)}{\Delta T}$$

The log of a complex number $z=r \angle \theta$ is $\ln(r) + j\theta$

If $B$ is nonsingular (the situation here) then the generalized eigenvalues are the eigenvalues of $B^{-1}A$
Matrix Pencil Method with Many Signals

• The Matrix Pencil approach can be used with one signal or with multiple signals

• Multiple signals are handled by forming a $Y_k$ matrix for each signal $k$ using the measurements for that signal and then combining the matrices

$$Y_k = \begin{bmatrix}
    y_{1,k} & y_{2,k} & \cdots & y_{L+1,k} \\
    y_{2,k} & y_{3,k} & \cdots & y_{L+2,k} \\
    \vdots & \vdots & \ddots & \vdots \\
    y_{m-L,k} & y_{m-L+1,k} & \cdots & y_{m,k}
\end{bmatrix}$$

$$Y = \begin{bmatrix}
    Y_1 \\
    \vdots \\
    Y_N
\end{bmatrix}$$

The required computation scales linearly with the number of signals.
Matrix Pencil Method with Many Signals

• However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals are actually need to be included to determine the desired modes

• Ultimately we are finding

\[ y_j(t_j, \alpha) = \sum_{i=1}^{n} b_i \phi_i(t_j, \alpha) \]

• The \( \alpha \) is common to all the signals (i.e., the system modes) while the \( b \) vector is signal specific (i.e., how the modes manifest in that signal)
Quickly Determining the $b$ Vectors

- A key insight is from an approach known as the Variable Projection Method (from Borden, 2013) that for any signal $k$

$$y_k = \Phi(\alpha)b_k$$

And then the residual is minimized by selecting $b_k = \Phi(\alpha)^+ y_k$

where $\Phi(\alpha)$ is the $m$ by $n$ matrix with values

$$\Phi_{ji}(\alpha) = e^{\alpha_i t_j} \text{ if } \alpha_i \text{ corresponds to a real eigenvalue,}$$

and $\Phi_{ji}(\alpha) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j)$ and $\Phi_{ji+1}(\alpha) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)$

for a complex eigenvalue; $t_j = (j-1)\Delta T$

Finally, $\Phi(\alpha)^+$ is the pseudoinverse of $\Phi(\alpha)$

Where $m$ is the number of measurements and $n$ is the number of modes

Iterative Matrix Pencil Method

When there are a large number of signals, the iterative matrix pencil method works by:

- Selecting an initial signal to calculate the $\alpha$ vector
- Quickly calculating the $b$ vectors for all the signals, and getting a cost function for how closely the reconstructed signals match their sampled values
- Selecting a signal that has a high cost function, and repeating the above adding this signal to the algorithm to get an updated $\alpha$

An open access paper describing this is W. Trinh, K.S. Shetye, I. Idehen, T.J. Overbye, "Iterative Matrix Pencil Method for Power System Modal Analysis," Proc. 52nd Hawaii International Conference on System Sciences, Wailea, HI, January 2019; available at scholarspace.manoa.hawaii.edu/handle/10125/59803
Texas 2000 Bus Synthetic Grid Example

- This synthetic grid serves an electric load on the ERCOT footprint (the grid itself is fictional)
- We’ll use the Iterative Matrix Pencil Method to examine its modes
  - The contingency is the loss of two large generators

The measurements will be the frequencies at all 2000 buses
2000 Bus System Example, Initially Just One Signal

- Initially our goal is to understand the modal frequencies and their damping
- First we’ll consider just one of the 2000 signals; arbitrarily I selected bus 8126 (Mount Pleasant)
Some Initial Considerations

- The input is a dynamics study running using a $\frac{1}{2}$ cycle time step; data was saved every 3 steps, so at 40 Hz
  - The contingency was applied at time = 2 seconds
- We need to pick the portion of the signal to consider and the sampling frequency
  - Because of the underlying SVD, the algorithm scales with the cube of the number of time points (in a single signal)
- I selected between 2 and 17 seconds
- I sampled at ten times per second (so a total of 150 samples)
2000 Bus System Example, One Signal

- The results from the Matrix Pencil Method are verified.
Some Observations

- These results are based on the consideration of just one signal
- The start time **should** be at or after the event!

If it isn’t then…

The results show the algorithm trying to match the first two flat seconds; this should not be done!!
2000 Bus System Example, One Signal Included, Cost for All

- Using the previously discussed pseudoinverse approach, for a given set of modes (\( \alpha \)) the \( b_k \) vectors for all the signals can be quickly calculated.

\[
b_k = \Phi(\alpha)^+ y_k
\]

- The dimensions of the pseudoinverse are the number of modes by the number of sample points for one signal.

- This allows each cost function to be calculated.

- The Iterative Matrix Pencil approach sequentially adds the signals with the worst match (i.e., the highest cost function).
2000 Bus System Example, Worst Match (Bus 7061)
2000 Bus System Example, Two Signals

With two signals

The new match on the bus that was previously worst (Bus 7061) is now quite good!

With one signal

The new match on the bus that was previously worst (Bus 7061) is now quite good!
2000 Bus System Example, Iterative Matrix Pencil

- The Iterative Matrix Pencil intelligently adds signals until a specified number is met
  - Doing ten iterations takes about four seconds
Takeaways So Far

• Modal analysis can be quickly done on a large number of signals
  – Computationally is an $O(N^3)$ process for one signal, where $N$ is the number of sample points; it varies linearly with the number of included signals
  – The number of sample points can be automatically determined from the highest desired frequency (the Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency)
  – Determining how all the signals are manifested in the modes is quite fast!!
Getting Mode Details

- An advantage of this approach is the contribution of each mode in each signal is directly available.

This slide shows the mode with the lowest damping, sorted by the signals with the largest magnitude in the mode.
Visualizing the Modes

• If the grid has embedded geographic coordinates, the contributions for the mode to each signal can be readily visualized.

Image shows the magnitudes of the components for the 0.63 Hz mode; the display was pruned to only show some of the values.
Application to a Larger System

• The following few slides show an application to a larger, 110 bus real system modeling a proposed ac interconnection of the North American Eastern and Western grids.

• Takeaway from the project is there are no show stoppers to doing this though if the grids are interconnected, there should be more than a few interconnection points (we studied nine)
WECC Frequency Comparison: With and Without the AC Interconnection

The graph compares the frequency response for three WECC buses for a severe contingency with the interface (thick lines) and without (thin lines).
Bus Frequency Results for a Generator Outage Contingency

Image shows the frequencies at all 110,000 buses; it was run for 80 seconds just to demonstrate the system stays stable.

For modal analysis we’ll be looking at the first 20 second
Spatial Frequency Contour
(Movies Can Also be Easily Created)

This visualization is using geographic data views and a contour to show the response of the 110,000 bus model; red values are frequencies less than 60 Hz.
Iterative Matrix Pencil Method Applied to 43,400 Substation Signals

Processing all 43,400 signals took about 75 seconds (with 20 seconds of simulation data, sampling at 10 Hz)
Iterative Matrix Pencil Method Applied to 43,400 Substation Signals

Verifying the Results

Matching for a large deviation example

The worst match (out of 43,400 signals); note the change in the y-axis
Large System Visualization of a Mode using Geographic Data Views
Summary

• The webinar has covered the power system application of measurement-based modal analysis
• Techniques are now available that can be readily applied to both small and large sets of power system measurements, either from the actual system or from simulations
• The result is measurement-based modal analysis is now be a standard power system analysis tool
• Large-scale system results can also be readily visualized
Questions?
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Prepublication copies of papers can be downloaded at overbye.engr.tamu.edu/publications (with paper 3 from 2021 [and its references] a good place to start)