Digital Algorithms Based on Traveling Waves

- Introduction
  - Line model for traveling waves
  - Forward and backward waves
    - Reflections and Measurement
  - Traveling wave algorithms
    - Single phase, Three phase
  - Traveling wave algorithms
    - Directional, Distance, Differential
Characteristics of Digital Algorithms

Model
- Lumped Parameter
  - Domain
    - Time
    - Frequency
  - Result
    - Distance to the fault
    - Direction of the fault
    - Place of the fault
- Traveling Wave

Model Form
- Model
  - Response to some signal component
- Conversion to digital form (derivative)

Residual
- Neglected
  - Direct Solutions
  - Approximate Integrals
  - Approximate Derivative
- Optimized
  - MLSE Techniques
- Optimization Techniques
  - Recursive
    - Kalman Filtering
  - Non Recursive
    - MLSF Technique
    - Linear Estimation
  - Convolution
  - Correlation
- Approximate
  - Approximate Derivative
  - Approximate Integrals

©2014 Mladen Kezunovic. All rights reserved
Introduction
Digital Algorithms Based on Traveling Waves

- Introduction
- **Line model for traveling waves**
  - Forward and backward waves
    - Reflections and Measurement
  - Traveling wave algorithms
    - Single phase, Three phase
- Traveling wave algorithms
  - Directional, Distance, Differential
Line model for traveling waves

Telegrapher equations:

\[- \frac{\partial v(x, t)}{\partial x} = l \cdot \frac{\partial i(x, t)}{\partial t} \]
\[- \frac{\partial i(x, t)}{\partial x} = c \cdot \frac{\partial v(x, t)}{\partial t} \]

Solution:

\[v(x, t) = F_1(x - Vt) + F_2(x + Vt)\]
\[i(x, t) = \frac{1}{z} [F_1(x - Vt) - F_2(x + Vt)]\]

Traveling wave model \((r = 0, g = 0)\)

Example:

\[v(t) = z \left\{ F \left( t - \frac{x}{v} \right) + f \left( t + \frac{x}{v} \right) \right\}\]
\[i(t) = F \left( t - \frac{x}{v} \right) - f \left( t + \frac{x}{v} \right)\]

where

\[V = \frac{1}{\sqrt{l \cdot c}}\] Surge Velocity
\[Z = \sqrt{\frac{l}{c}}\] Surge Impedance

©2014 Mladen Kezunovic. All rights reserved
Line model for traveling waves: Digital algorithms

1. What is the distance between the relay and the fault? (Distance algorithms)

2. What is the direction of the fault? (Direction algorithms)

3. Is there a leakage of the current between two relays? (Differential algorithms)
Digital Algorithms Based on Traveling Waves

- Introduction
- Line model for traveling waves
- **Forward and backward waves**
  - Reflections and Measurement
- Traveling wave algorithms
  - Single phase and Three phase
- Traveling wave algorithms
  - Directional, Distance and Differential
Forward and Backward Waves: Back Ground

Voltage shape along the line:

Forward Wave $F_1(x-vt_o)$ at $t = t_o$

Backward Wave $F_2(x+vt_o)$

$v \to v$ at $t = t_o + \Delta t$, $x_o = v \cdot \Delta t$

$\to x_1 \to x_2 \to x$
Forward and Backward Waves: Back Ground

Current shape along the line:

\[ F_1(x - Vt_o) \]

\[ t = t_o \]

\[ t = t_o + \Delta T \]

\[ -F_1(x + Vt_o) \]
Forward and Backward Waves: Reflections (An example)

The Reflected Forward Wave Becomes a Backward One

Note:
Reflections are multiplied by $K_F$. $K_F$ is the reflection coefficient and it depends on the nature of the discontinuity.
Forward and Backward Waves: Measurement

Sending End: \( x = 0, \quad v(0,t) = v_S(t) \)
\( i(0,t) = i_S(t) \)

\[ v_S(t) = F_1(Vt) + F_2(\bar{V}t) \]
\[ i_S(t) = \frac{F_1(-Vt)}{z} - \frac{F_2(Vt)}{\bar{z}} \]

\[ 2F_1(-V \cdot t) = v_S(t) + z \cdot i_S(t) \]
\[ 2F_2(V \cdot t) = v_S(t) - zi_S(t) \]
Forward and Backward Waves: Measurement

Receiving End: \( x = d, \quad v(d, t) = v_R(t) \)
\( i(d, t) = -i_R(t) \)

\[
v_R(t) = F_1(d - Vt) + F_2(d + Vt)
\]
\[
-i_R(t) = \frac{F_1(d - Vt)}{z} - \frac{F_2(d + Vt)}{z}
\]

\[
2F_1(d - Vt) = v_R(t) - z i_R(t)
\]
\[
2F_2(d + Vt) = v_R(t) + z i_R(t)
\]
Forward and Backward Waves: Bergeron’s Equations

\[ 2F_1(-Vt) = v_S(t) + Zi_S(t), \quad \tau = \frac{d}{V} \]

\[ t \rightarrow t - \tau \]

\[ 2F_1(d - Vt) = v_S(t - \tau) + Zi_S(t - \tau) \]

\[ 2F_1(d - Vt) = V_R(t) - Zi_R(t) \]

1st Bergeron’s Equation:

\[ v_S(t - \tau) + Zi_S(t - \tau) = v_R(t) - Zi_R(t) \]
Forward and Backward Waves: Bergeron’s Equations

\[ 2F_2(Vt) = v_S(t) - z i_S(t) \quad t \to t + \tau \]
\[ 2F_2(d + Vt) = v_S(t + \tau) - z \cdot i_S(t + \tau) \]
\[ 2F_2(d + Vt) = v_R(t) + z \cdot i_R(t) \]

2nd Bergeron’s Equation

\[ v_S(t + \tau) - z i_S(t + \tau) = v_R(t) + z i_R(t) \]

or

\[ v_S(t) - z i_S(t) = v_R(t - \tau) + z i_R(t - \tau) \]
Forward and Backward Waves: Superimposed Values

Pre-Fault Values: \( v_S(t), i_S(t), v_R(t), i_R(t) \) (Harmonics)

Post Fault Values: \( \hat{v}_S(t) + \Delta v_S(t), \hat{i}_S(t) + \delta i_S(t) \)
\( \hat{v}_R(t) + \Delta v_R(T), \hat{i}_R(t) + \Delta i_R(t) \)

\( \hat{v}_S(t), \hat{i}_S(t), \hat{v}_R(t), \hat{i}_R(t) \) are pre-fault values extended to the future.

\( \Delta v_S(t), \Delta i_S(t), \Delta v_R(t), \Delta i_R(t) \) are caused by the fault, and are calculated using superimposed line.
Digital Algorithms Based on Traveling Waves

- Introduction
- Line model for traveling waves
- Forward and backward waves
  - Reflections and Measurement

**Traveling wave algorithms**
- Single phase and Three phase

**Traveling wave algorithms**
- Directional, Distance and Differential
Traveling Wave Algorithms

- Single phase consideration
- Three phase consideration
  - Three phase equations
    \[- \frac{\partial [u]}{\partial x} = [L] \frac{\partial [i]}{\partial t} \]
    \[- \frac{\partial [i]}{\partial x} = [c] \frac{\partial [u]}{\partial t} \]
  - Decoupling of equations into single-phase representations
  - Modal analysis
Traveling Wave Algorithms: Three Phase Considerations

Three phase equations:

\[ \begin{align*}
- \frac{\partial U(x,t)}{\partial t} &= L \frac{\partial I(x,t)}{\partial x} \\
- \frac{\partial I(x,t)}{\partial x} &= C \frac{\partial U(x,t)}{\partial t}
\end{align*} \]

\( U(x,t) \), \( I(x,t) \) – phase voltage and currents

\( L \quad C \) Matrices

Problem: We need decoupled equations.

Modal Transformation:

\[ \begin{align*}
V(x,t) &= SV_{\text{mode}}(x,t) \\
I(x,t) &= QI_{\text{mode}}(x,t)
\end{align*} \]

S and Q Transformation matrices

©2014 Mladen Kezunovic. All rights reserved
Traveling Wave Algorithms: Solution of the Eigen Value Problem

Problem

\[ Ax = y \]

The problem is to find \( x \) (eigenvector) so that

\[ Ax = \lambda x \]

where \( \lambda \) are the eigen values

Solution

\[ [S]^{-1}[L][Q] = [L_{\text{mode}}] \]

If \([S]^{-1}\) is a matrix of eigenvectors of \([L]\) then \([L_{\text{mode}}]\) is a diagonal matrix of eigenvalues of \([L]\).
Traveling Wave Algorithms: Transformation Matrices

- **General case**
  
  \( S \) and \( Q \) are determined from the solution of Eigen value problem for matrices \( L \cdot C \) and \( C \cdot L \)

- **Transposed line**
  
  \( S \) and \( Q \) do not depend on \( L \) and \( C \).

There are more solutions:

\[
S = Q = \begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix}
\]

\[
a = e^{j \frac{2\pi}{3}}
\]

\[
S = Q = \begin{bmatrix}
1 & 1 & 1 \\
2 & -1 & -1 \\
0 & \sqrt{3} & -\sqrt{3}
\end{bmatrix}
\]

\[
S = Q = \begin{bmatrix}
1 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2
\end{bmatrix}
\]
Digital Algorithms Based on Traveling Waves

- Introduction
- Line model for traveling waves
- Forward and backward waves
  - Reflections and Measurement
- Traveling wave algorithms
  - Single phase and Three phase

**Traveling wave algorithms**
- Directional, Distance and Differential
Traveling Wave Algorithms

- Direction algorithms
  - Signs of superimposed values
  - The sequence of forward and backward waves

- Distance algorithms
  - Time of travel of a reflected forward wave

- Differential algorithms
  - Check of Bergeron’s equation validity
Traveling Wave Algorithms: Example of a Directional algorithm

- $i_{fs}(t)$ is for the Relay a Backward Wave
- When the fault is in front of the relay, backward wave appears first, and then a forward wave follows.
Traveling Wave Algorithms: Example of a Directional algorithm

Criterion:
If both relays situated on the Line’s end see the fault “in front”, the fault is in the line.

©2014 Mladen Kezunovic, All rights reserved
Traveling Wave Algorithms: Example of a Directional algorithm

- Question: How we register that some wave picks-up?

- Possibilities
  - Measure $F_1(t)$ and $F_2(t)$
  - Measure
    \[
    [F_1(t)]^2 + \left[ \frac{1}{\omega} \cdot \frac{\partial}{\partial t} F_1(t) \right]^2
    \]
    \[
    [F_2(t)]^2 + \left[ \frac{1}{\omega} \cdot \frac{\partial}{\partial t} F_2(t) \right]^2
    \]

- Problem: Two relays must exchange their conclusions: A communication link is needed.
Traveling Wave Algorithms: Example of a Distance Relay

Some Wave Generated Behind the Relay Arrives to the Relay

$t_1 = t_0$, Relay Registers a Forward Wave

$t_2 = t_0 + \tau$, Wave arrives at the Fault and the Reflection Starts

$t_3 = t_0 + 2\tau$, The same wave arrives at the Relay as Backward Wave

\[
\tau_x = \frac{t_3 - t_1}{2}, \quad x = \frac{\tau_x}{V}
\]

\[
\phi'(\tau) = \int_{t_1}^{t_3} \left[ F_1(t) - \overline{F_1}(t) \right] \left[ F_2(t + \tau) - \overline{F_2}(t + \tau) \right] dt
\]

Bar denotes time average $\phi'(\tau)$ is maximal if $\tau = 2\tau_x$
Traveling Wave Algorithms: Example of a Differential Relay

If there is no fault, then two Bergeron’s equations hold:

\[ \xi(t) = v_S(t - \tau) + z_i_S(t - \tau) - v_R(\tau) + z \cdot i_R(\tau) = 0 \]

\[ \zeta(\tau) = v_X(t) - z \cdot i_S(t) - v_R(t - \tau) - z \cdot i_R(t - \tau) = 0 \]

if \( \xi(t) \neq 0 \) or \( \zeta(t) \neq 0 \) There is a fault

NOTE: \( \xi(t) \) and \( \zeta(t) \) are delayed fault currents \( i_{fS}(t) \) and \( i_{fR}(t) \)

Problem: An exchange of data between two relays is needed.