Characteristics of Digital Algorithms

- **Domain**
  - Time
  - Frequency

- **Model**
  - Lumped Parameter
  - Traveling Wave

- **Result**
  - Distance to the fault
  - Direction of the fault
  - Place of the fault

- **Model Form**
  - Differential Equation

- **Residual**
  - Neglected
  - Optimized

- **Conversion to digital form (derivative)**
  - Approximate Integrals
  - Approximate Derivative
  - Correlation
  - Convolution

- **Signal Processing Techniques**
  - Non Recursive
  - Recursive

- **Optimization Techniques**
  - MLSF Technique
  - Linear Estimation

- **Optimized**

- **Neglected**

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Digital Algorithms

• Lumped Parameter Model
  - Time Domain Approach
  - Frequency Domain Approach

• Traveling Wave Model
  - Directional Algorithm
  - Distance Algorithm
  - Differential Algorithm
Digital Algorithms: Line Models

1. Lumped Parameter Model

- Time Domain Model (Signals are functions of time)

Example:
\[ u(t) = Ri(t) + L \frac{di(t)}{dt} \]

- Frequency Domain Model
  (Signals are represented with transforms)

Example:
\[ v(jw) = RI(jw) + jwL \cdot I(jw) \]

Question: What is the distance \( x \) between the relay and the fault? (Distance Algorithms)

\[ x = \frac{L}{l} \]
Lumped Parameter Model
Time Domain Approach

Model Equation: \[ u(t) = Ri(t) + L \frac{di(t)}{dt} + e(t) \]

• Treatment of the Derivative Term:
  - Approximation Using Samples
  - Integration of the Equation Using Samples

• Treatment of the Residual Term
  - Residual Term is neglected
  - Influence of the Residual Term is Minimized

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Treatment of the Derivative

\[ u(t) = Ri(t) + L \frac{di(t)}{dt} + e(t) \]

Problem: Samples of \( u(t) \) and \( i(t) \) can be measured, but for \( \frac{di(t)}{dt} \) is not possible.

Question: How to approximate \( \frac{di(t)}{dt} \) at \( t = k\Delta t \) with samples of \( i(t) \) denoted \( i_k \)
Approximations of current derivative

Backward: \[ \frac{i_k - i_{k-1}}{\Delta t} \]

Forward: \[ \frac{i_{k+1} - i_k}{\Delta t} \]

Middle: \[ \frac{i_{k+1} - i_{k-1}}{2\Delta t} \]
Approximations of current derivative
(continued)

How was it derived?

\[ i_k = A \sin(kw_0 \Delta t + \phi) \]

\[ i_{k-1} = A \sin(kw_0 \Delta t - w_0 \Delta t + \phi) = \]

\[ A \sin(kw_0 \Delta t + \phi) \cos(w_0 \Delta t) - A \sin(kw_0 \Delta t + \phi) \sin(w_0 \Delta t) \]

\[ \frac{di(t)}{dt} = w_0 A \cos(kw_0 \delta t + \phi) = \frac{i_k \cos(w_0 \Delta t) - i_{k-1}}{\sin(w_0 \Delta t)} w_0 \]

If \( w_0 \Delta t \to 0 \), we get backward approximatuion
Integration of the Differential Equation

Model Equation: \[ u(t) = Ri(t) + L \frac{di(t)}{dt} + e(t) \]

\[
\int_{t_1}^{t_2} u(t) dt = R \int_{t_1}^{t_2} i(t) dt + L \left[ i(t_2) - i(t_1) \right] + \int_{t_1}^{t_2} e(t) dt
\]

Problem: How to approximate \( \int_{t_1}^{t_2} x(t) dt \) with samples \( x_k \)?

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General Lumped Parameter Digital Model of the Line

\[ J^u_m = R \cdot J^i_m + L \cdot J^{i'}_m + J^e_m \]

\[ J^u_m, J^i_m, J^{i'}_m \] Linear Combination of Delayed Signal Samples

\[ J^e_m \] Immeasurable Residual Term

\[ m \] Present moment

Example 1: Use of forward approximation

\[ u_m = R \cdot i_m + L \frac{i_m - i_{m-1}}{\Delta t} + e_m \]

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Examples 2: Use of approximate integration

\[
\sum_{k=m-N}^{m-1} u_k \Delta t = R \cdot \sum_{k=m_N}^{m-1} i_k \Delta t + L \left[ i_m - i_{m-N} \right] + \sum_{k=m_N}^{m-1} e_k \Delta t
\]

Problem: How to treat the residual term?
Treatment of Residual Term

- Direct Solution (Ignore The Residual Term)

Calculate $J_{m1}^u, J_{m1}^i, J_{m1}^{i'}$ for two moments $m1$ and $m2$, and then solve for estimates $\hat{R}$ and $\hat{L}$ two equations:

$$J_{m1}^u = \hat{R} \cdot J_{m1}^i + \hat{L} \cdot J_{m1}^{i'}$$

$$J_{m2}^u = \hat{R} \cdot J_{m2}^i + \hat{L} \cdot J_{m2}^{i'}$$
• Minimum Least Square Technique

- Calculate \( m_1, m_2, \ldots, m_r \)
- Find \( \hat{R} \) and \( \hat{L} \) that minimize

\[
\sum_{m=m_1}^{m_r} (J^e_m)^2
\]

\[
J^e_m = J^u_m - \hat{R}J^i_m + \hat{L}J^{i'}_m
\]

\( J^u_m, J^i_m, J^{i'}_m \) Linear Combination of Delayed Signal Samples
\( J^e_m \) Immeasurable Residual Term
\( m \) Present moment
# Time Domain Approach Algorithms

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Frequency Domain Approach

• Parameter estimation with no optimization:
  - Signal Contains Only the First Harmonics
  - Signal Contains Other Harmonics

• Parameter Estimation With Optimization
  - Least Square Fit
  - Linear Estimation
  - Kalman Filtering
Lumped Parameter Model

Frequency Domain Approach

• Initial Model:

\[ u(t) = V \cos(w_0 t + \phi) + \sum_k C^u_k f^u_k + n_v(t) \]
\[ i(t) = I \cos(w_0 t + \varphi) + \sum_k C^i_k f^i_k + n_i(t) \]

Where:

- \( C^u_k, C^i_k, V, I, \phi, \gamma \) - unknown coefficients (Signal Parameters)
- \( f^u_k(t), f^i_k(t) \) - known functions representing higher harmonics and transients
- \( n_u(t), n_i(t) \) - noise terms
Lumped Parameter Model
Frequency Domain Approach (continued)

• Impedance is calculated as:

\[ z = R + jw_0L = \frac{V}{I} \cos(\phi - \varphi) + j \frac{V}{I} \sin(\phi - \varphi) \]

• Signal Parameters to be estimated are:

\[ V, I, \phi, \gamma \]

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Two Approaches to the Signal Parameter Estimation

1. The noise term and some signal components are neglected. Parameters are obtained by correlation and convolution signals.

   Examples of Simplified Models:
   - The signal contains the fundamental harmonic only
   - The signal is represented with Fourier series having a finite number of higher harmonics

2. The estimate of the parameters are found so that they are optimal in some sense.

   Examples of Techniques:
   - Least Square Method
   - Linear Estimation
   - Kalman Filtering
The output $y(t)$ is equal to the area of the window.

$$y_n \approx \sum_{k=n-N}^{n} x_{n-k} W_{n-k} \Delta t$$
Signal Convolution

\[ y_n \equiv \sum_{k=n-N}^{n} x_{n-k} W_k \Delta t \]

The output \( y(t) \) is equal to the area in the window.
Examples of Correlation and Convolution

-Correlation

DIRECT FOURIER ANALYSIS

• $W(t)$ are $\sin(w_0 t)$ and $\cos(w_0 t)$. Length of window is equal to the period $T$ or a half a period $T/2$.

INDIRECT FOURIER ANALYSIS

• $W(t)$ are Haar or Walsh functions. The output is used to find Fourier coefficients.

CORRELATION FUNCTION METHOD

• $W(t)$ is current or voltage. The length of the window is $T$ or $T/2$.

-Convolution

Most commonly the weight functions is fundamental frequency sinusoid
Fourier Based Methods

- Signal Model:  $x(t) = \sum_{m=0}^{r} a_m \cos mwt + \sum_{m=1}^{r} b_m \sin mwt$

$$a_1 = X_R \quad b_1 = X_I$$

Example:

Fourier decomposition of a square function $y_s$: 1st, 3rd and 5th harmonics after summarizing generate $y'$ (dotted line)
Fourier Based Methods  (continued)

• Direct Fourier Model:
  \[ X_R(t_1) = \frac{1}{T} \int_{t_1-T}^{t_2} x(t) \cos(\omega t) dt \]
  \[ X_I(t_1) = \frac{1}{T} \int_{t_1-T}^{t_2} x(t) \sin(\omega t) dt \]

• Indirect Fourier Model:
  \[ x(t) = \sum_{i=1}^{M} k_i f_i(t) \]

  \( f_i \) – set of orthogonal functions:
  • Walsh
  • Haar

Find \( k_i \) by correlation then express \( X_R \) and \( X_I \) using \( k_i \)
Example of Indirect Fourier Analysis

Walsh Function Matrix

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Walsh Functions

\[ W_s^0(t) \]
\[ W_s^1(t) \]
\[ W_s^2(t) \]
\[ W_s^3(t) \]
\[ W_s^4(t) \]
\[ W_s^5(t) \]
\[ W_s^6(t) \]
\[ W_s^7(t) \]
Walsh Functions  (continued)

Orthogonally of Walsh functions:

\[
\int_0^T W_R^k(t) W_P^m(t) \, dt = \begin{cases} 
T_0 & m = k \\
0 & m \neq k 
\end{cases}
\]

\[ R = 2^p \quad p = \text{integer} \]

Approximation with Walsh Functions

\[ x(t) \equiv \sum_{k=0}^{R-1} C_R^k W_R^k(t) \]

\[ 0 \leq t \leq T \]

\[ C_R^k = \frac{1}{T} \int_0^{T_0} x(t) W_R^k(t) \, dt \]
Signal Contains Only the First Harmonic

- Signal Model

\[ x(t) = X_R \cos(wt) + X_I \sin(wt) \]

- Example

\[
X_R = \frac{x_n \sin(n - 1)\delta - x_{n-1} \sin n\delta}{\sin \delta} \quad \delta = w\Delta t
\]

\[
X_I = \frac{x_{n-1} \cos n\delta - x_n \cos(n - 1)\delta}{\sin \delta}
\]
Signal Contains Only the First Harmonic (continued)

\[ R = \frac{U_R I_R + U_I I_I}{I_R^2 + I_I^2} \]

\[ = \frac{u_n i_n - u_n i_{n-1} \cos \delta - u_{n-1} i_n \cos \delta + u_{n-1} i_{n-1}^2}{i_n^2 - 2i_n i_{n-1} \cos \delta + i_{n-1}^2} \]

\[ \omega_o L = \frac{U_R I_I - U_I I_R}{I_R^2 + I_I^2} \]

\[ = -\frac{u_{n-1} i_n \sin \delta - i_{n-1} u_n \sin \delta}{i_n^2 - 2i_n i_{n-1} \cos \delta + i_{n-1}^2} \]
Optimal Parameter Estimation

- Least Square Fit -

• Signal Model

\[ x(t) = X_R \cos \omega_0 t + X_I \sin \omega_0 t + \sum_{k=1}^{r} C_k f_k(t) + e(t) \]

• Condition

\[ J(X_R, X_I, C_k) = \int_{t=T_0}^{t} e^2(t) dt \text{ minimum} \]

• Calculate values of \( \hat{X}_R \) and \( \hat{X}_I \) from equations:

\[ \frac{\partial J}{\partial X_R} = 0 \quad \frac{\partial J}{\partial X_I} = 0 \]

\[ \frac{\partial J}{\partial C_k} = 0 \quad k = 1 \ldots r \]
Parameter Estimation with Optimization

-Linear Estimation-

- Signal Model: \( x(t) = X_R \cos w_0 t + X_I \sin w_0 t + \sum_{k=1}^{r} C_k f_k(t) + e(t) \)

- Expression for the estimates:

\[
\hat{X}_R = \int_{t-T_0}^{t} A_R(t)x(t)dt \quad \hat{X}_I = \int_{t-T_0}^{t} A_I(t)x(t)dt
\]

- Problem: Find \( A_R(t) \) and \( A_I(t) \) such that

\[
E\left\{ \hat{X} - X \right\} = 0 \quad \text{and} \quad E\left\{ (\hat{X} - X)^2 \right\} = 0 \quad \text{is minimal}
\]

- Observations:

  - if \( e(t) \) is Gaussian, Linear Estimation becomes the Least Square Fit
  - if \( e(t) \) is Gaussian White Noise, \( f_k(t) \) and \( T_0 = T/2 \), then the technique is equal to Half-Cycle Fourier Transform
Kalman Filtering

- Basic Notes: 
  \[
  \begin{bmatrix}
  V(n) \\
  V(n+1)
  \end{bmatrix}
  \quad \text{state estimate}
  \]

- State Equations: 
  \[
  \begin{bmatrix}
  V(n+1)
  \end{bmatrix} = \begin{bmatrix}
  P
  \end{bmatrix} \begin{bmatrix}
  V(n)
  \end{bmatrix} + \begin{bmatrix}
  Q
  \end{bmatrix} \Delta V(n)
  \]

- Measurement: 
  \[
  \begin{bmatrix}
  V_s(n)
  \end{bmatrix} = \begin{bmatrix}
  C
  \end{bmatrix} \begin{bmatrix}
  V(n)
  \end{bmatrix} + \begin{bmatrix}
  b(n)
  \end{bmatrix}
  \]

  \[
  \Delta V(n), [b(n)] \quad \text{white noises}
  \]
Kalman Filtering (continued)

\( P, Q - \) covariance matrices of \([V(n)]\) and \([\Delta V(n)]\). Depending on components that are anticipated at input, covariance matrices can have different forms:

\[
[P] = [Q] = \begin{bmatrix}
\cos(\omega \Delta T) & -\sin(\omega \Delta T) \\
\sin(\omega \Delta T) & \cos(\omega \Delta T)
\end{bmatrix}
\]

\[
[P] = [Q] = \begin{bmatrix}
\cos(\omega \Delta T) & -\sin(\omega \Delta T) & 0 \\
\sin(\omega \Delta T) & \cos(\omega \Delta T) & 0 \\
0 & 0 & e^{-\lambda t}
\end{bmatrix}
\]

\[
[P] = [Q] = \begin{bmatrix}
\cos(\omega \Delta T) & -\sin(\omega \Delta T) & 0 & 0 \\
\sin(\omega \Delta T) & \cos(\omega \Delta T) & 0 & 0 \\
0 & 0 & \cos(2\omega \Delta T) & -\sin(2\omega \Delta T) \\
0 & 0 & \sin(\omega \Delta T) & \cos(2\omega \Delta T)
\end{bmatrix}
\]

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Kalman Filtering-Recursive Equations (continued)

We calculate in every step new (improved) values of $V_k$, $P_k$, $C_k$ starting from the initial values $X_0$ and $P_0$

\[
[K(n)] = \frac{[M(n)][C]^T}{[C][M(n)][C]^T + [B]}
\]

\[
[Z(n)] = [I - [K(n)][C]] \cdot [M(n)]
\]

\[
[M(n+1)] = [P][Z(n)][P]^T + [Q][U][Q]^T
\]

\[
[^{\hat{V}}(n)] = [P][^{\hat{V}}(n-1)] + [K(n)][V_S(n) - [C][P][^{\hat{V}}(n-1)]]
\]
# Frequency Domain Approach Algorithms

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Analysis of the Algorithm Properties

• Results should be available in a short time after the fault inception

• Mean should be equal to actual value

• Standard deviation should not be significant
## Algorithm Properties

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<td>Noise term</td>
<td>Sensitivity may be reduced</td>
<td>Sensitivity may be reduced</td>
</tr>
<tr>
<td><strong>Signal measurement</strong></td>
<td></td>
<td></td>
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<tr>
<td>Analog signal processing</td>
<td>Imp. response the same for voltages and currents</td>
<td>System function the same for $\omega_0$</td>
</tr>
<tr>
<td>Synchronization required</td>
<td>Optimal sampling freq. likely to exist</td>
<td>Non conclusion about optimal sampling freq.</td>
</tr>
<tr>
<td><strong>Data window</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of samples</td>
<td>Either depends on the sampling frequency or is not determined by any particular requirement</td>
<td>Fixed</td>
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</table>
# Class I characteristics

<table>
<thead>
<tr>
<th>Algorithm Characteristics</th>
<th>Class I</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elimination $\frac{di}{dt}$ term</td>
</tr>
<tr>
<td></td>
<td>Via samples</td>
</tr>
<tr>
<td><strong>Signal content</strong></td>
<td></td>
</tr>
<tr>
<td>Fundamental harmonic</td>
<td></td>
</tr>
<tr>
<td>Higher harmonic and transients</td>
<td></td>
</tr>
<tr>
<td>Noise term</td>
<td>Sensitive</td>
</tr>
<tr>
<td><strong>Signal measurement</strong></td>
<td></td>
</tr>
<tr>
<td>Analog signal processing</td>
<td>Imp. response the same for voltages and currents</td>
</tr>
<tr>
<td>Signal Sampling</td>
<td>Synchronization required</td>
</tr>
<tr>
<td></td>
<td>Optimal sampling freq. likely to exist</td>
</tr>
<tr>
<td>Major alg.</td>
<td></td>
</tr>
<tr>
<td>Data window</td>
<td>Can be: fixed, selected in optimal way, determined by the fixed number of samples and the sampling frequency, free to be selected</td>
</tr>
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<td>Either depends on the sampling frequency or is not determined by any particular requirement</td>
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# Algorithm Properties

<table>
<thead>
<tr>
<th>Algorithm Characteristics</th>
<th>Class I</th>
<th>Class II</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Elimination dV/dt term</td>
<td>Treatment of the e(t) term</td>
</tr>
<tr>
<td></td>
<td>Via samples</td>
<td>Via integration</td>
</tr>
<tr>
<td>Signal content</td>
<td>Theoretically do not influence the results</td>
<td>Some alg are not sensitive</td>
</tr>
<tr>
<td>Fundamental harmonic</td>
<td>Sensitive</td>
<td>Sensitivity may be reduced</td>
</tr>
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<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td>Imp. response the same for voltages and currents</td>
<td>System function the same for ( \omega ).</td>
</tr>
<tr>
<td>Signal measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sampling</td>
<td>Synchronization required</td>
<td>Synchronization is not required if the rotation is applied</td>
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<tr>
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<tr>
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<td>Can be: fixed, selected in optimal way, determined by the fixed number of samples and the sampling frequency, free to be selected</td>
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## Class II characteristics

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>No optimization</td>
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<td></td>
<td>First Harmonic</td>
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<tr>
<td>Fundamental harmonic</td>
<td>Some alg. are not sensitive</td>
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<tr>
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</tr>
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<td>Data window</td>
</tr>
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<td>Number of samples</td>
</tr>
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</table>
Analysis of the Algorithm Properties

- Signal Contents
- Signal Measurements
- Major Algorithm Parameters

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Signal Contents

- Fundamental Harmonic

- Higher Harmonics and Transients
  - Results are more accurate when these signal components Die-Off

- Noise Term
  - The accuracy depends on the treatment of the Noise Term
Fundamental Harmonic

- Two groups of algorithm sense the change differently
- Parameter L is sensitive to frequency change
- Frequency change, in general, affects algorithms of the second group
Signal Measurements

- Analog Signal Processing
  - Conditions for the use of the filtered signals
  - The role of the antialiasing filter
  - Time Delay of the analog filter

- Signal Sampling
  - Sampling Synchronization
  - Sampling Frequency

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Major Algorithm Parameters

- Data Window
- Number of Samples
Data Window

• Fixed

• Can be selected in an optimal way

• Determined by the fixed number of samples and the sampling frequency

• Free to be selected
Number of Samples

- Fixed
- Depends on the sampling frequency
- Not determined
## Summary

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