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New Digital Signal Processing Algorithms for Frequency Deviation Measurement

M. Kezunović, P. Spasojević
Department of Electrical Engineering
Texas A&M University
College Station, Tx 77843-3128

B. Peruničić
Elektrotehnički Fakultet
71000 Sarajevo,
Yugoslavia

Abstract: This paper introduces two digital signal processing algorithms for frequency deviation measurement. The algorithms are derived using a new signal processing scheme based on quadratic forms of signal samples. These algorithms provide high measurement accuracy over a wide range of frequency changes. One is designed for measurements of small deviations of the nominal frequency whereas the other one measures off-nominal frequency deviations. Performance of the algorithms is evaluated using computer simulation tests.

Keywords: Frequency measurements, Digital algorithms, Signal processing, Quadratic forms.

INTRODUCTION

This paper is concerned with two issues: the small frequency deviation measurements and the off-nominal frequency deviation measurements. Frequency information is one of the most important parameters for system monitoring and control. Load shedding, load restoration, generator protection from overspeeding and detection of the generation-load out-of-step conditions may in general be based on the small frequency deviation measurements. For generators, the over-excitation detection and the voltage and current estimates during start-up and shut-down procedures may be based on the off-nominal frequency deviation measurements.

Previous work in this field has resulted in a variety of algorithms for the small frequency deviation measurements. Some of these algorithms use known signal processing techniques such as Discrete Fourier Transform, Least Error Squares, and Kalman Filtering [1-5], while others use a heuristic approach [6, 7]. Most of them use the sinusoidal model for the signal. Their efficiency (accuracy) is influenced by one or more of the following factors: superimposed noise, non-linear static characteristic, and slow response. In general, increased accuracy and robustness require greater complexity. The query for more accurate, computationally simple and robust algorithms continues.

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The off-nominal frequency deviation measurement algorithms were proposed in two references [8, 9]. Both use the results of a small frequency deviation algorithm and calculate the required correction. One is using the phase-locked loop and the other one is using a look-up table for correcting the estimate. Both of these methods are relatively complex and slow.

This paper presents two algorithms for measurement of frequency deviations. The first algorithm is the result of an attempt to overcome the deficiences in the small frequency deviation algorithms stated above. The approach was to look into existing frequency measurement algorithms for a common expression that could be utilized for the development of an accurate estimate of the frequency deviation [10]. The new process of determining the coefficients of this general form has resulted in a simple and accurate algorithm.

The second algorithm was designed to capture a wide range of the off-nominal frequency deviations by adapting (extending) the frequency deviation algorithm developed by the authors in an earlier reference [10]. Only three more multiplications were introduced to achieve the improved accuracy. The algorithm remains extremely simple, fast and accurate. No table-looking and no iterations are necessary to calculate the correction.

Extensive testing was performed with both algorithms. Static and dynamic tests show high accuracy and fast response. Algorithm robustness was tested using additive noise tests and Electromagnetic Transient Program (EMTP) network simulation tests. Both algorithms performed well by not amplifying the noise and converging fast for the EMTP generated signal transients.

The general expression used to derive both algorithms was recognized earlier by the authors as being suitable for accurate measurements of a number of power system quantities [11-15]. This leads the authors to a conclusion that a custom designed signal processing chip may be developed to implement the generalized algorithm form. Selection of the appropriate coefficients may enable use of the same chip for various applications such as frequency deviation, line parameter, and power measurements.

First part of the paper gives theoretical background and the frequency algorithm design procedure. Second part outlines derivation of the new algorithm for measurements of the small frequency deviations. Third part presents a new, very accurate and extremely simple algorithm for off-nominal frequency deviation measurements. Fourth part provides results of the extensive testing performed using both algorithms.

NEW DSP APPROACH

This section presents theoretical background and design procedure of a Digital Signal Processing (DSP) approach used to derive new algorithms.

It starts by introducing a general algorithm expression. This expression is recognized to be a common form for most of the previously introduced non-recursive frequency deviation measurement algorithms. The general algorithm expression is a quotient of quadratic forms (QQF) of signal samples. This expression is made constant in time and real for all frequency deviations of a sinusoidal signal. This is achieved by imposing constraints on the coefficients of the quadratic forms. Finally, the Taylor expansions of the quadratic forms give a quotient of two frequency deviation polynomials whose coefficients can be chosen so that the quotient becomes equal to the value of the signal frequency deviation.

The second subsection gives the steps of a formal procedure for the design of algorithms for frequency deviation measurements. This procedure shows a way of deriving the quadratic forms' coefficients satisfying the performance requirements and the constraints derived in the first subsection.

Theoretical Background

The following quotient of quadratic forms of signal samples is recognized to be a general form for frequency deviation measurement algorithms [10]:

$$\begin{aligned}\Delta\hat{f} &= \frac{\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}\overline{a}_{km}x_{n-k}x_{n-m}}{\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}b_{km}x_{n-k}x_{n-m}} \\ &= \frac{KF\overline{A}}{KFB} \qquad (1)\end{aligned}$$

KF are the quadratic forms, $\overline{a}_{km}$ and b_{km} form matrices $\overline{\mathbf{A}}$ and $\mathbf{B}$ associated with quadratic forms. Their definition is given in APPENDIX A.

Let us assume the following input signal representation:

$$\begin{aligned}x(t) &= X\cos(\omega t+\phi) \qquad (2)\\ x_n &= X\cos(\omega n\Delta t+\phi)\\ &= X\cos(nd+\phi) \qquad (3)\end{aligned}$$

Electrical angle deviation Δd is proportional to the deviation of frequency since $\Delta d = \Delta\omega\cdot\Delta t = 2\pi\Delta f\cdot\Delta t$.

For simplicity, we shall use the electrical angle deviation estimate which is of the same form as the frequency deviation estimate given in the equation (1):

$$\begin{aligned}\Delta\hat{d} &= \frac{\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}a_{km}x_{n-k}x_{n-m}}{\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}b_{km}x_{n-k}x_{n-m}} \\ &= \frac{KFA}{KFB} \qquad (4)\end{aligned}$$

where $a_{km} = \overline{a}_{km}/(2\pi\Delta t)$.

Conditions for expression (4) to be constant and to give the value of the signal frequency deviation are derived as follows.

In the case of a sinusoidal waveform, given by equation (3), the value of a quadratic form is:

$$\begin{aligned}KF(n) &= \sum_{k=0}^{N-1}\sum_{m=0}^{N-1}h_{km}X^2\cos[(n-k)d+\phi]\cos[(n-m)d+\phi]\\ &= \frac{X^2}{2}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}h_{km}\cos[(m-k)d]\\ &+ \frac{X^2}{2}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1}h_{km}\cos\{[2n-(k+m)]d+2\phi]\}\\ &= KF^c + KF^v(n) \qquad (5)\end{aligned}$$

Equation (5) shows that the value of the quadratic form, in the case of a sinusoidal signal, consists of a constant KF^c and an oscillating $KF^v(n)$ component.

The constant part KF^c, can be expressed in the following way [11]:

$$KF^c = \frac{X^2}{2}Re\{\mathbf{K}^c(e^{-jd})\} \qquad (6)$$

where:

$$\begin{aligned}\mathbf{K}^c(w) &= \sum_{r=-N+1}^{N-1}h_r^c w^r\\ h_r^c &= \sum_k\sum_m \underset{k-m=r}{h_{km}}\\ w &= e^{-jd}\end{aligned}$$

Also, the variable part $KF^v(n)$, can be expressed as follows [11]:

$$KF^v(n) = \frac{X^2}{2}|\mathbf{K}^v(e^{-jd})|\cos\{arg[\mathbf{K}^v(e^{-jd})]+2nd+2\phi\} \quad (7)$$

where:

$$\begin{aligned}\mathbf{K}^v(w) &= \sum_{r=0}^{2N-2}h_r^v w^r\\ h_r^v &= \sum_k\sum_m \underset{k+m=r}{h_{km}} \qquad (8)\\ w &= e^{-jd}\end{aligned}$$

Since for a steady state sinusoidal signal frequency deviation is constant, the oscillating component KF^v should be identical to zero for all frequency deviations and for all n. It can easily be seen from the equation (8) that $KF^v(n)$ is identical to zero when the following conditions are satisfied:

$$\begin{aligned}h_r^v &= 0 \qquad (9)\\ r &= 0,\ldots,2N-2\end{aligned}$$

From the equation (9) one can see that these conditions are equivalent to saying that the sums of the elements h_{km} on the anti-diagonal and all the sub-anti-diagonals of the quadratic form matrix $\mathbf{H}$ are equal to zero.

For the constant component KF^c to be real for all frequencies, it is sufficient for the quadratic form matrix to be symmetrical. For a symmetric matrix, the following holds:

$$\mathbf{K}^c(w) = h_0^c + 2\sum_{r=0}^{N-1}h_r^c Re\{w^r\} = Re\{\mathbf{K}^c(w)\} \qquad (10)$$

where $w = e^{-jd}$.

Equations (9) and (10) produce a constant and real value for the quadratic form of equation (5) as follows:

$$KF(n) = h_0^c + 2\sum_{r=0}^{N-1} h_r^c Re\{w^r\} = \sum_{r=0}^{N-1} h_r \cos(rd) \qquad (11)$$

Therefore, the QQF for frequency deviation measurements, given by equation (1), may now be expressed as follows:

$$\Delta\hat{d} = \frac{\sum_{r=0}^{N-1} a_r \cos(rd)}{\sum_{r=0}^{N-1} b_r \cos(rd)} = \frac{\mathbf{A}(d)}{\mathbf{B}(d)} \qquad (12)$$

where:

$$a_0 = \sum_k \sum_{m\ k=m} a_{km}$$
$$b_0 = \sum_k \sum_{m\ k=m} b_{km}$$
$$a_r = \sum_k \sum_{m\ k-m=r, r\neq 0} a_{km}$$
$$b_r = \sum_k \sum_{m\ k-m=r, r\neq 0} b_{km}$$

This form is constant and real for all deviations of the electrical angle. In order to obtain estimates of the electrical angle, functions **A** and **B** are expressed in the forms of their Taylor expansions. As a result, the following general expression for frequency deviation estimate is obtained:

$$\Delta\hat{d} = \frac{\mathbf{A}(d_0) + \mathbf{A}'(d_0)\Delta d + \mathbf{A}''(d_0)\frac{(\Delta d)^2}{2} + \cdots}{\mathbf{B}(d_0) + \mathbf{B}'(d_0)\Delta d + \mathbf{B}''(d_0)\frac{(\Delta d)^2}{2} + \cdots} \qquad (13)$$

Equation (13) is the generic algorithm expression used in the next section to derive different algorithms.

Algorithm Design Procedure

The algorithm design procedure is related to the appropriate selection of coefficients in the generic algorithm expression given by equation (13). The algorithm is designed according to the application constraints. The following steps allow for the design of the algorithms for frequency deviation measurements.

- The following algorithm parameters are chosen: data window, number of signal samples, sampling frequency, measurement range, and degree of desired accuracy.
- The size of the quadratic forms' matrices is chosen using first three parameters.
- Last two algorithm parameters are used for selecting the order of the polynomials in the equation (13). Having selected the polynomial order, necessary constraints need to be imposed on the remaining coefficients. Incidentally, the most general set of conditions for the equation (13) to give the signal frequency deviation estimate can be determined.
- Two sparse symmetric quadratic forms' matrices are formed. The nonzero elements of these matrices are then selected to satisfy equation (9). Also, in order to satisfy the conditions derived in the third step, the total number of nonzero elements needs to be greater or equal to the number of these conditions.
- A frequency deviation algorithm is derived by solving the equations obtained in the third step for the unknown elements of the matrices formed in the fourth step.
- Last, noise sensitivity and the accuracy requirements are checked. If they are not satisfied one goes back to the fourth step to change the matrix elements. If necessary, one may also go back to the third step to select a different order for the polynomials.

SMALL FREQUENCY DEVIATION MEASUREMENT ALGORITHM

The described algorithm design procedure is implemented as follows.

If one chooses a data window length of three quarters of the nominal period, and a sampling rate of sixteen samples per cycle, then the nominal electric angle for this case is equal to $d_0 = \frac{\pi}{8}$.

To achieve high accuracy four terms of the Taylor expansions are used here. In this case, for equation (13) to give the signal frequency deviation, the most general set of conditions set upon the remaining polynomial coefficients is as follows:

$$\begin{aligned} \mathbf{A}(d_0) &= 0 \\ \mathbf{B}'''(d_0) &= 0 \\ \mathbf{A}'(d_0) &= \mathbf{B}(d_0) \\ \frac{\mathbf{A}''(d_0)}{2} &= \mathbf{B}'(d_0) \\ \frac{\mathbf{A}'''(d_0)}{3} &= \mathbf{B}''(d_0) \end{aligned} \qquad (14)$$

In this case, the frequency deviation estimate given by equation (13) approximates the signal frequency deviation as follows:

$$\Delta\hat{d} \approx \Delta d \qquad (15)$$

The goal is to determine the quadratic forms' coefficients. Accordingly, the fourth step of the frequency deviation measurement algorithm design procedure requires forming two sparse symmetric matrices whose sums of elements on the anti-diagonal and all the sub-anti-diagonals are equal to zero (follows from equation (9)). Another requirement is that these matrices have a greater (or equal) number of nonzero elements than the number of algorithm coefficients' constraints derived in the third step of the procedure.

Matrices of the following form are used:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & a & b & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 & d \\ 0 & -2a & -b & 0 & 0 & 0 & -c & 0 & 0 & 0 & 0 & -d & 0 \\ a & -b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & e & f & 0 & 0 & 0 & g \\ 0 & -2e & -f & 0 & 0 & 0 & -g & 0 \\ e & -f & 0 & 0 & 0 & 0 & 0 & 0 \\ f & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -g & 0 & 0 & 0 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix **B** is a symmetric 13 by 13 matrix that can be reduced to an 8 by 8 matrix because the remaining elements are zero. Both matrices satisfy the above requirements. They have in total seven nonzero elements against the five algorithm coefficients' constraints given by equation (14). Two of these elements are assumed in a way that simplifies the calculation of other elements and enables getting small enough numbers for the other elements.

For matrices of this form, and for $c = 0.05$ and $d = 0.01$, coefficient constraints, expressed by equations (14), give as a result the following values for the coefficients of quadratic forms: $a = -1.044303503155235$, $b = 0.52823862829380$, $e = 0.78543321540189$, $f = -1.05813678372674$ and $g = -0.03443924221845$.

The estimate of the electrical angle deviation, given by equation (4), now becomes:

$$\Delta\hat{d} = \frac{x_n \cdot w_0 - x_{n-1} \cdot w_1}{x_n \cdot z_0 - x_{n-1} \cdot z_1} \tag{16}$$

where:

$$\begin{aligned} w_0 &= a \cdot x_{n-2} + b \cdot x_{n-3} + c \cdot x_{n-7} + d \cdot x_{n-12} \\ w_1 &= a \cdot x_{n-1} + b \cdot x_{n-2} + c \cdot x_{n-6} + d \cdot x_{n-11} \\ z_0 &= e \cdot x_{n-2} + f \cdot x_{n-3} + g \cdot x_{n-7} \\ z_1 &= e \cdot x_{n-1} + f \cdot x_{n-2} + g \cdot x_{n-6}. \end{aligned}$$

Results presented in the algorithm testing section indicate high accuracy and low noise sensitivity of the obtained algorithm.

OFF-NOMINAL FREQUENCY MEASUREMENT ALGORITHM

Due to their simplicity, algorithms for small frequency deviation measurement do not calculate the off-nominal frequency deviation of the sinusoidal signal very accurately. As shown in reference [9], one can calculate the measurement error in an off-line mode. This error estimate can be used to improve the frequency deviation estimate.

The goal of this paper is to introduce an efficient and simple measurement algorithm for off-nominal frequency deviations. The Taylor expansion of the estimate error function is used to derive this new algorithm.

To design an algorithm that uses a data window which is equal to one half of the nominal period and a sampling rate of eight samples per cycle, one can proceed as follows. In this case the nominal electric angle is equal to $d_0 = \frac{\pi}{4}$.

Let us assume low influence of the Taylor expansion terms in equation (13) that are higher than the third order. To get an accurate estimate of the electric angle deviation, the following constraints for the coefficients of the frequency deviation polynomials are imposed.

$$\begin{aligned} \mathbf{A}(d_0) &= 0 \\ \mathbf{B}''(d_0) &= 0 \\ \frac{\mathbf{A}''(d_0)}{2} &= \mathbf{B}'(d_0) \\ \mathbf{A}'(d_0) &= \mathbf{B}(d_0) \end{aligned} \tag{17}$$

In order to simplify the selection of the quadratic forms' coefficients, some of the coefficients are assumed to be zero. For the reasons given earlier, quadratic forms' matrices are chosen to be symmetrical and to have the sums of the elements on the antidiagonal and all sub-antidiagonals equal to zero.

Hence, the matrices have this form:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & a & b & c \\ 0 & -2a & -b & 0 & 0 \\ a & -b & -2c & 0 & 0 \\ b & 0 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & g & f & h \\ 0 & -2g & -f & 0 & 0 \\ g & -f & -2h & 0 & 0 \\ f & 0 & 0 & 0 & 0 \\ h & 0 & 0 & 0 & 0 \end{bmatrix}$$

Furthermore, for $f = 0$ and $g = 1$, equations (17) give as a result the following values for the coefficients of quadratic forms: $a = 0.5$, $b = 0$, $c = -0.25$ and $h = 0$.

After the selection of the quadratic forms' coefficients, equation (4) gives as a result the following algorithm for determining the electrical angle deviation estimate:

$$\Delta\hat{d} = 0.5 \cdot \left[1 - 0.5 \cdot \frac{x_n \cdot x_{n-4} - x_{n-2}^2}{x_n \cdot x_{n-2} - x_{n-1}^2}\right] \tag{18}$$

For a sinusoidal signal $x = X\cos(nd + \phi)$, equation (12) gives:

$$\begin{aligned} \Delta\hat{d} &= \frac{-0.5 + \cos(2d) - 0.5\cos(4d)}{2 \cdot [cos(2d) - 1]} \\ &= \frac{\sin(2\Delta d)}{2}, \quad d = d_0 + \Delta d \end{aligned} \tag{19}$$

This equation is much simpler than the one given in reference [9]. Following the approach given in reference [9], a more accurate estimate for off-nominal frequency deviation can be obtained. This is achieved by using an arcsin look-up table which helps to solve the following equation:

$$\Delta d = \frac{\arcsin(2 \cdot \Delta\hat{d})}{2} \tag{20}$$

A different approach that obviates the need for a look-up table and gives a more simple solution is described here.

Five terms of the arcsine Taylor expansion series provide the following corrected electrical angle estimate:

$$\begin{aligned} \Delta\hat{d} &= 0.5 \cdot \left[1 - 0.5 \cdot \frac{x_n \cdot x_{n-4} - x_{n-2}^2}{x_n \cdot x_{n-2} - x_{n-1}^2}\right] \\ \Delta\hat{d}_1 &= \Delta\hat{d} \cdot \Delta\hat{d} \\ \Delta\hat{d}_2 &= \Delta\hat{d}_1 \cdot \Delta\hat{d}_1 \\ \Delta\hat{d}_{corr} &= \Delta\hat{d} \cdot \left(1 + \frac{\Delta\hat{d}_1}{6} + 3 \cdot \frac{\Delta\hat{d}_2}{40}\right) \end{aligned} \tag{21}$$

Only three new estimate multiplications are introduced. In this way the algorithm remains extremely simple and easy to implement.

Test results given below show a great improvement in the static as well as in the dynamic accuracy of the developed algorithms. Improved dynamic and static accuracy for the off-nominal frequency deviation algorithm, and lower noise sensitivity of the small deviation algorithm are achieved by the new algorithm designs.

TEST RESULTS

The algorithms are tested using a synthesized sinusoidal signal and a voltage signal output from Electromagnetic Transient Program (EMTP) [16].

Three tests were performed using a synthesized sinusoidal signal. First, algorithm static accuracy was tested. The second test evaluated algorithm dynamic response. Third, algorithm noise sensitivity for a simulated data acquisition and signal processing system, was tested.

Transient test was performed using simulation of a fault and a load disturbance in the test system. The EMTP voltage output was used as the algorithm input signal.

Static Test

In this test, synthesized sinusoidal signals with frequencies in the range from 40 to 80 Hz in steps of 1 Hz were provided as inputs to the algorithms. Results given in Figure 1 show a comparison of the algorithm outputs. High measurement accuracy may be observed.

Dynamic Test

Frequency deviation algorithms were applied to a synthesized sinusoidal signal with an oscillating and decreasing frequency. This resembles the system frequency change in the event of power deficiency in a power system. The following equation shows the change in frequency over time:

$$f(t) = 60 - 10 \cdot t - 1 \cdot \sin(2\pi \cdot 5t) \tag{22}$$

Results in Figure 2 show a very good dynamic response of the algorithms within their range of accuracy.

Noise Test

A simple data acquisition and signal processing system is considered for the noise test. This system consists of a 12 bit analog-to-digital converter, low pass filter and one of the measurement algorithms. A sinusoidal 60 Hz signal with superimposed white zero-mean Gaussian noise was used as input for the test. The test block diagram is shown in Figure 3.

The results for the Small Deviations (SD) Algorithm and the Off-Nominal Deviations (OND) Algorithm are shown in Table 1. This table shows the relation between the mean and standard deviation of the noise and the mean and standard deviation of the relative measurement error. Since the algorithms have a very high static and dynamic accuracy within the frequency range from 54 to 66 Hz, one can assume a measurement range of 12 Hz. This range has been used for calculating the mean and standard deviation value of the relative measurement error . Results indicate that the algorithms in this system configuration do not amplify the noise and that they do not introduce a bias. They also show that the SD algorithm is slightly less influenced with noise than the OND algorithm.

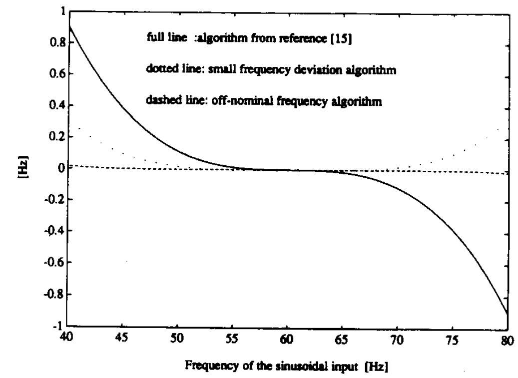

Fig. 1. Frequency Deviation Error for the Three Algorithms for the Static Test

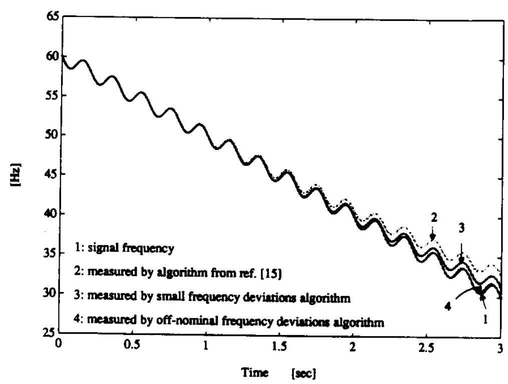

Fig. 2. Dynamic Test Results for the Three Algorithms

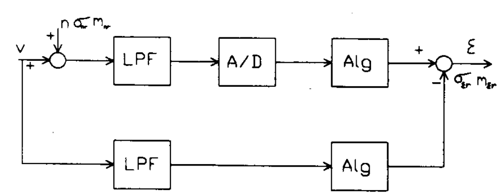

Fig. 3. Block Diagram for the Noise Test

where:

v is a synthesized sinusoidal waveform.

n is a white zero mean Gaussian noise.

σ_{nr} is the relative standard deviation of the noise.

m_{nr} is the mean value of the noise.

σ_{er} is the relative standard deviation of the measurement error.

m_{er} is the mean value of the measurement error.

LPF is Butterworth low pass filter of the fourth order with 76.8 Hz cutoff frequency.

A/D is a 12 bit analog to digital converter.

Alg is a frequency deviation algorithm.

Table 1: Noise Test Results

Meas. Number	Noise Characteristics		Output Error Characteristics	
SD alg.	m_{nri}	σ_{nri}	m_{eri}	σ_{eri}
1	-0.0059	0.0986	0.0016	0.0824
2	0.0017	0.0498	$-4.121 \cdot 10^{-4}$	0.0447
3	$-2.06 \cdot 10^{-4}$	0.0102	$-1.719 \cdot 10^{-4}$	0.0111
OND alg.				
1	-0.0046	0.1033	$-1.5 \cdot 10^{-4}$	0.1129
2	$1.75 \cdot 10^{-4}$	0.0504	$-1.07 \cdot 10^{-4}$	0.0573
3	$-7.3 \cdot 10^{-5}$	0.099	$-2.025 \cdot 10^{-4}$	0.0119

Transient Test

Two EMTP simulations were performed. Both simulations used a model of a synchronous machine with an exciter and a governor. For the first simulation, a simple two transmission line model shown on Figure 4 was modeled. For the second simulation, system model shown on Figure 5 was used. Line Z_1 in Figure 4 and line Z_c in Figure 5 were modeled using the EMTP distributed-parameter line model. Line Z_2 in Figure 4 was modeled using the EMTP lumped-parameter line model. Node voltage output v, from the first and the second EMTP simulation, were used as inputs for the Small Deviations (SD) Algorithm and the Off-Nominal Deviations (OND) Algorithm, respectively.

For the first simulation, that used a model shown in Figure 4, a 20 milliseconds long load disturbance was applied. For the second simulation, that used a model shown in Figure 5, a three phase fault at 30 milliseconds as well as a fault clearance at 100 milliseconds were applied.

Figure 6 shows a block diagram of the measurement scheme used for testing the two algorithms.

Three tests have been performed with both algorithms. First test calculates the frequency deviation directly from the EMTP output. Second test uses a low pass filter (LPF) for filtering the EMTP output. Third test uses both LPF and the following scheme for averaging the frequency estimate:

$$\Delta \hat{f}_n^{av} = \frac{1}{8} \cdot \sum_{k=0}^{7} \Delta \hat{f}_{n-k} \tag{23}$$

Test results are shown in figures 7 to 12. Figures 7 to 9 show deviation estimates calculated by the Small Deviations (SD) Algorithm. Results for the three test conditions are indicated together with the scaled change in synchronous machine velocity. Figures 10 to 12 show the same estimates for the Off-Nominal Deviations (OND) Algorithm. One can see that SD Algorithm follows the rate of change of the small frequency deviation, after filtering the input or averaging the estimate, quite accurately. The OND Algorithm follows large off-nominal deviations of frequency very accurately. In general, the transient tests performed by the authors, and not reported here, show that the SD algorithm follows more closely the small frequency deviations, whereas the OND algorithm follows more closely the large off-nominal deviations.

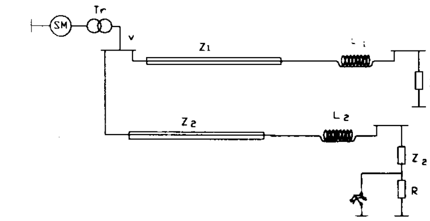

Fig. 4. EMTP System Model for Testing the Small Deviations (SD) Algorithm

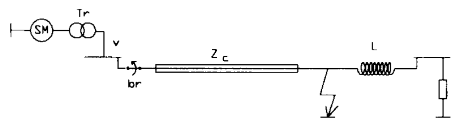

Fig. 5. EMTP System Model for Testing the Off-Nominal Deviations (OND) Algorithm

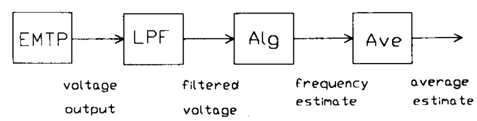

Fig. 6. Testing Scheme Using EMTP Output

LPF - Butterworth low pass filter of the fourth order

Alg - frequency deviation algorithm

AVE - eight-sample averaging unit

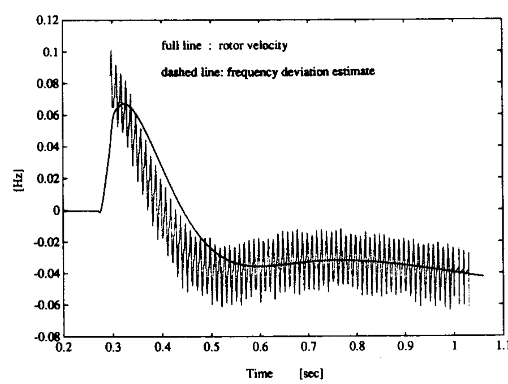

Fig. 7. SD Algorithm Estimate

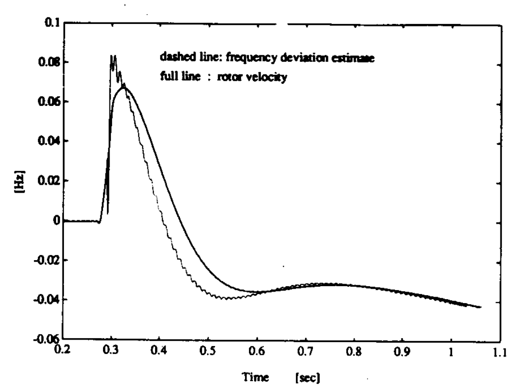

Fig. 8. SD Algorithm Estimate with Voltage Filtering

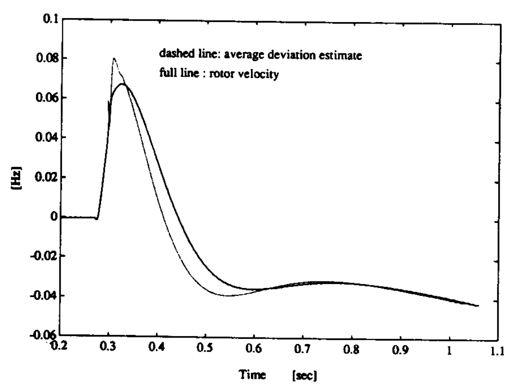

Fig. 9. SD Algorithm Average Estimate with Voltage Filtering

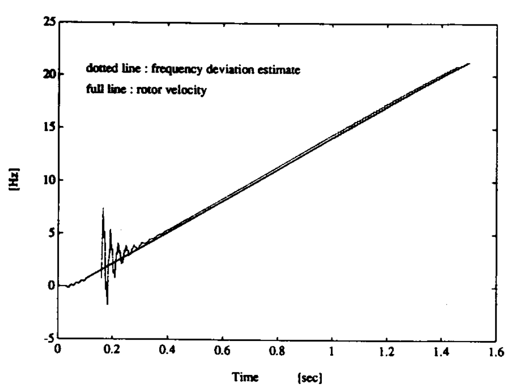

Fig. 10. OND Algorithm Estimate

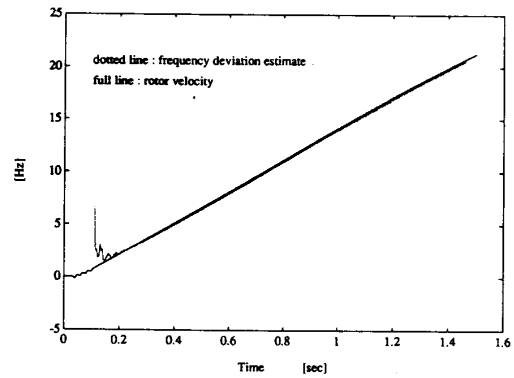

Fig. 11. OND Algorithm Estimate with Voltage Filtering

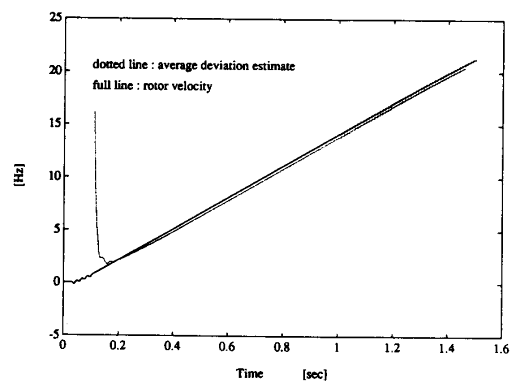

Fig. 12. OND Algorithm Average Estimate with Voltage Filtering

CONCLUSIONS

The results presented in the paper lead to the following conclusions:

- The two algorithms defined in this paper are extremely accurate and yet quite simple to implement.
- The new algorithm design approach enables definition of the generic algorithm form.
- The generic algorithm form provides a straight-forward way to define new algorithms for frequency deviation measurements.
- Algorithm implementation may be further optimized by developing a custom DSP chip that performs the calculation for a general quadratic form of signal samples.

ACKNOWLEDGEMENTS

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APPENDIX A

Quadratic forms of a signal x, denoted here as KF, may be expressed using matrix notation in the following way:

$$\begin{aligned} KF(n) &= \sum_{m=0}^{N-1}\sum_{k=0}^{N-1} b_{mk}x_{n-k}x_{n-m} \\ &= \mathbf{x}^T\mathbf{B}\mathbf{x} \end{aligned} \tag{24}$$

where:

$$\begin{aligned} \mathbf{x}^T &= [x_n\ x_{n-1}\ldots x_{n-N+1}] \\ \mathbf{B} &= \{b_{km}\} \end{aligned}$$

x_n is a signal sample at the discrete time n, $\mathbf{B}$ is the quadratic form matrix and b_{km} are the elements of this matrix.

Mladen Kezunović (S'77, M'80, SM'85) received his Dipl. Ing. degree from University of Sarajevo, Yugoslavia, the MS and PhD degrees from University of Kansas, all in electrical engineering in 1974, 1977 and 1980, respectively.

Dr. Kezunović's industrial experience is with Westinghouse Electrical Corporation in the USA, and the Energoinvest Company in Yugoslavia. He also worked at University of Sarajevo, Yugoslavia. He was a Visiting Associate Professor at Washington State University and at Texas A&M University, for the 1986-1987 and 1987-1989 academic years, respectively.

Dr. Kezunović's current research interest is in the areas of digital algorithms for relaying and control, and advanced product development for power system automation.

Predrag Spasojević received his Dipl. Ing. degree from University of Sarajevo, Yugoslavia in Electrical Engineering in 1989. Currently, he is a graduate student at the Department of Electrical Engineering, Texas A&M University, working towards his MS degree in the area of digital signal processing applications in power systems.

Branislava Peruničić received her Dipl Ing. degree in Electrical Engineering from the University of Belgrade, Yugoslavia, and Candidate of Sciences degree in automatic control from the Institute for Automatics and Telemechanics of USSR Academy of Sciences, Moscow in 1960, and 1969, respectively.

From 1960 up to now she is with Energoinvest Company, Sarajevo, Yugoslavia. In 1964 she also joined the faculty of the University of Sarajevo, Yugoslavia, where she is now a Professor of Electrical Engineering. Dr. Peruničić work has been in automatic control, and electric power system control and planning. Her current fields of interest are graph theory, variable structure systems, and applications of signal processing.

Discussion

I. Kamwa and R. Grondin (Hydro-Québec, IREQ, Varennes, Canada) : For some time, the discussers have been involved in designing algorithms and devices for real-time estimation of power system quantities, such as phasor and frequency [A]. Thus, they have followed the authors' work [11][12][15] with much interest. The bilinear form approach to the synthesis of signal processing algorithms seems a quite powerful tool, and its application to the measurement of small frequency deviations and off-nominal frequencies has been described with great care in the present paper. The two resulting algorithms compare favorably with some previous schemes [9], in terms of both accuracy and computational complexity.

However, the authors' comments about the following concerns could enhance the clarity of the paper.

1/ The frequency estimates in fig. 7 show an oscillating component. When the raw EMTP voltage is filtered through the low pass filter, the interfering component is smoothed out (fig. 8). At first sight, we believe that the noise in fig. 7 results from a 2nd harmonic in the phase voltage, since by the frequency conversion process, synchronous machine are known to generate such components during transient periods [B]. Thus, when the 2nd harmonic is *filtered out* (or more exactly, *greatly attenuated*) by the Butterworth filter, the phase voltage becomes a so-called "noise-free" signal, and the algorithm performs well. This behaviour, even if suitable, raises a question: since harmonics, subharmonics and DC trends are unavoidable in actual power system measurements, are we right to conclude that filtering of the phase voltage is really necessary for the algorithm to provide useful frequency estimates? In this context, we wonder if a band-pass filter, centered around the line-frequency, is not preferable to a simple low-pass, since DC offset and subharmonics eventually present in the phase voltage should also be attenuated. As shown in [A], this can be succesfully achieved using a digital band-pass FIR filter designed through DFT.

Along the same lines, we find the results in table 1 somewhat discouraging since they show that a 1% noise on the phase voltage roughly converts to a 1% error on the frequency estimates. Thus, a 1% subharmonic distortion (in a series compensated network for example), would induced on the line-frequency estimates, a subharmonic jitter of nearly 60mHz. Perhaps the authors should comment on wether this represents some misunderstanding in our part.

2/ A large part of our work on frequency estimation has been oriented towards power system stabilizer (PSS) applications [C]. We know from experience that, not only the accuracy as measured through the experiment in fig. 1 is an important factor, but also that the phase lag at frequency modulation below 10Hz, can be of great concern [D]. When the phase lag introduced by the digital transducer is high (greater than that due for instance to a 40ms time constant), the estimates normally tend to be more accurate, but the stabilizer gain needs to be reduced, in order to maintain the feedback loop stable. When the maximum achievable gain is lower than that required by the planning division to warrant the overall power system stability, the usefulness of the stabilizer is greatly reduced and the only solution in this case is to use a better transducer or change the controller input signal. In this context, any delay in the measurement chain is a flaw which should be corrected whenever possible. The eight tap FIR filter used to smooth raw frequency estimates (figs. 9 and 12), is remarkable for its simplicity. But since it introduces a four sample pure delay on the estimates, this price can be too high for the achieved accuracy, at least for sensitive applications such as those described in [C] and [D].

3/ Our last point is a matter of detail: how do the authors explain that the estimated rotor speed leads the actual rotor speed (figs. 7, 8 and 10)? Since the algorithm as expressed by equations 16 and 21 relies on past samples only (up to lag 12), and does not make any explicit extrapolation of the future frequency value based on some high pass filtering equivalent, we feel that we need a little more information to understand fully such a behaviour.

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Manuscript received October 14, 1991.

M. KEZUNOVIC, P. SPASOJEVIC, B. PERUNICIC: The authors wish to thank the discussers for their interesting comments and questions.

Before attempting to respond to the points raised in the discussion, the authors feel that the following general comments are due:

- Any frequency measurement algorithm design may consist of three processing steps: input signal filtering, frequency (deviation) measurement, smoothing of the results.
- The filtering and smoothing algorithms are used to optimize the performance of the frequency measurement algorithm for a given application.
- Selection of the filtering and smoothing algorithms is usually driven by the following factors: dynamic characteristics of the input signal (voltage), computational behavior of the frequency measurement algorithm, time response and accuracy requirements of a given application, hardware/software implementation constraints.

Our development was concentrating on derivation of digital algorithms for two broad application areas, namely small and off-nominal frequency deviation measurements. An effort was made to derive algorithms that are very accurate, computationally simple and robust. To illustrate these achievements, a number of tests were performed and quite good results were obtained as shown in the paper. However, since the goal of the tests was not to demonstrate an optimized frequency measurement algorithm for a given application, the selection of the input signal filtering and the result smoothing algorithms was quite arbitrary. In particular, the selection of the filtering and the smoothing techniques given in the paper would not be the same for both the small and the off-nominal frequency deviation measurements if an optimized measurement of each was required.

Having the above stated general comments in mind, it is understandable that the use of the algorithms given in the paper in any specific application would require a thorough analysis of the requirements for all of the three processing steps before an optimized algorithm for that application is designed. The discussers have designed a frequency measurement algorithm for a very interesting application related to power system stabilizer control. Their comments relate to the references concerning mentioned application. Even though we have found the discussers' comments very interesting, our understanding of the mentioned application is limited at this time. Therefore, our replies can not be considered a final judgment regarding suitability of our algorithms for the mentioned application.

The following are our replies given in the order as stated by the discussers:

1/ The first comment relates to the selection of the input signal filtering. The discussers are inquiring about the input signal filtering characteristics used in Figures 7 and 8, giving only the results for the small deviation (SD) algorithm. As commented earlier, if a given application requires an accurate measurement of only the small frequency devatiation then the choice of the filtering may have to be different from what is given in our paper. The filter that the discussers have introduced in their reference [A] may be a very good choice. In order to illustrate the SD algorithm performance for a different choice of the input filtering, the SD algorithm estimate was obtained using the input filter specified by the discussers. The results are shown in Figure A. The corresponding results from the paper, shown in Figure 8, are repeated in Figure B for easy comparison. (A small difference between Figure 8 and Figure B is explained as a part of the comment #3.) Comparison of the results given in Figures A and B indicates that in this test case the SD algorithm performance does not change much with the use of the band-pass input filter instead of a low-pass filter.

The discussers have also raised a question if the filtering was really necessary for the algorithm to provide useful frequency estimates. In order to answer the question, one would need to study the application to determine if it is possible to characterize the occurence of DC offset, harmonics and subharmonics. If these components are present then the input signal filtering is needed. In that case a study of the application is needed to determine an optimum filter selection. However, if the DC offset, harmonic and subharmonic modes are known, the algorithm design methodology presented in the paper can be extended to accomodate rejection of the known modes [1]. In this case very simplified filtering may be sufficient, or filtering may not even be needed.

Finally the discussers are inquiring about the SD algorithm performance under the influence of the noise and the subharmonics. First, it should be noted from Figure 3 given in the paper that the noise is applied prior to Low Pass Filtering (LPF) and Analog to Digital (A/D) conversion. Therefore, the choice of these components will make a difference in the results. To illustrate this point, additional test results are shown in Table A. These results correspond to noise characteristics given in entry #3 (SD algorithm results) of Table 1, presented in the paper. Table A gives results of a sensitivity study where the influence of A/D and filtering selection was studied. The first three entries show results for different choices of A/D conversion setup, while the LPF block was the same as in the paper. It can be observed that the 16 bit A/D choice gives better results than the 12 bit one. Furthermore, two different band-pass filter designs were used. The subsequent two entries in Table A show the results when the LPF block was substituted with a band-pass filter, while the 12-bit A/D is removed. Again, the output error was further reduced. Finally, the last entry in Table A shows the results for a band-pass filter and a 16-bit A/D. These results are much better than the results obtained for a low-pass filter and a 12-bit A/D originally selected. Along these lines, it would not be correct to draw a conclusion about the algorithm error under the presence of the subharmonics based on the results given in Table 1. Furthermore, it would be hard to make any sound judgement at all as how the subharmonics affect the algorithm accuracy unless an optimized version of our algorithm is designed for this application and tested under the presence of subharmonics.

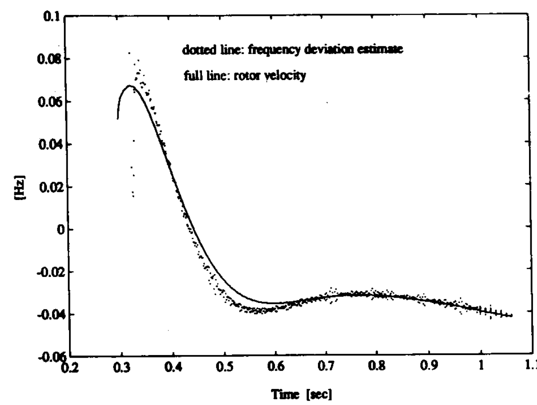

Fig. A. SD Algorithm Estimate with Band-pass Filtering

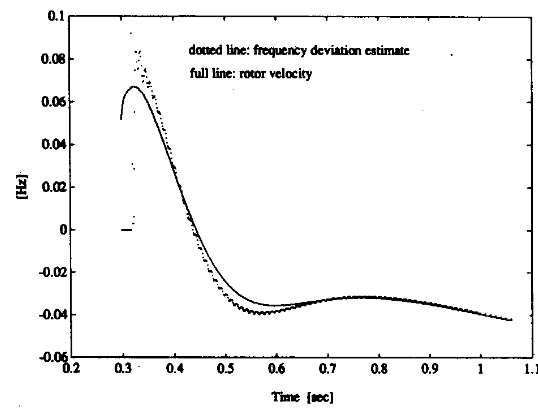

Fig. B. SD Algorithm Estimate with Low-pass Filtering

Table A. Additional Noise Test Results for the SD Algorithm

Meas. Parameters	Noise Characteristics		Output Error Characteristics	
A/D conv.	m_{nri}	σ_{nri}	m_{eri}	σ_{eri}
12 bit	$-2.06\cdot10^{-4}$	.0102	$-1.719\cdot10^{-4}$	.0111
16 bit	$9.78\cdot10^{-5}$	.01	$-1.08\cdot10^{-4}$	.0093
no	$-1.05\cdot10^{-6}$	.099	$-1.617\cdot10^{-4}$	.0081
Bandpass [Hz]	m_{nri}	σ_{nri}	m_{eri}	σ_{eri}
43.2-76.8	$-2.22\cdot10^{-4}$	.0101	-.0065	.0027
52.8-67.2	$1.16\cdot10^{-4}$	.0102	-.029	.0006
52.8-67.2 (16b)	$-1.29\cdot10^{-4}$	.0103	-.029	.0008

2/ This comment, as we understand it, relates to the trade off between the frequency measurement accuracy and the computational complexity of the three processing steps, namely input signal filtering, frequency measurement, and result smoothing. The discussers are inquiring about a time delay introduced by the smoothing algorithm given in the paper since this time delay may be critical in the application studied by the discussers. Again, the selection of the smoothing filter in the paper was quite arbitrary and did not take into account the specific application described by the discussers. It is the authors' opinion that an optimization of the frequency measurement algorithm for the mentioned application may produce much more computationally efficient solution while still preserving a remarkable accuracy. To illustrate this point, a result of the SD algorithm obtained by using the band-pass filter and a simplified one-pole smoother, introduced by the discussers in reference [A], is given in Figure C. Comparison of the results from Figure C with the results given in Figure B indicates that the overall algorithm accuracy is still preserved.

3/ The authors want to thank the discussers for indicating irregularities in Figures 7, 8, and 10. This is due to an oversight in the figure making process when the measurement curves in Figures 7 to 12 were mistakenly shifted to the left. When a correction is made (as given in Figure B for Figure 8) it can be observed that the overall results are even better since the estimates in Figure B, for example, follow the rotor velocity more closely than what is shown in Figure 8.

Since the discussers have inquired about the possible extrapolation effect due to the algorithm itself, an additional test was run to clarify the point. The algorithm has been tested using the sinusoidal input voltage signal whose frequency was changing sinusoidally around the nominal 60Hz. The input signal frequency change and the algorithm estimate are shown in Figure D. The results show that the estimate does not lead the input signal. However, it was interesting to observe that the estimate overshooting phenomenon is still present, as it is demonstrated in Figures 7 to 12 in the paper. It is our speculation that this is caused by the nonlinear processing that takes place during the evaluation of the frequency deviation estimate. Further study may be needed to clarify this phenomenon.

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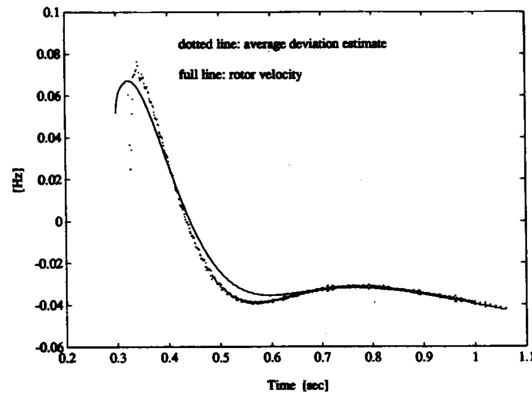

Fig. C. SD Estimate with Bandpass Filtering and Smoothing

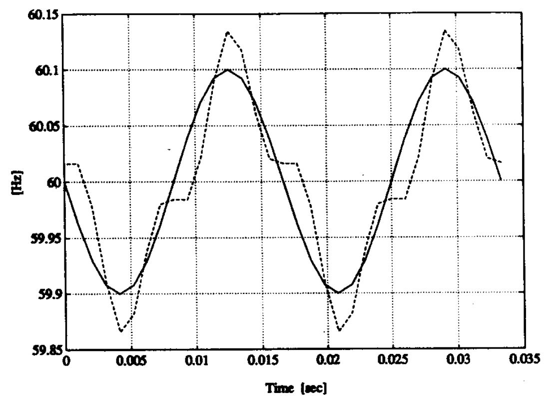

Fig. D. Sinusoidal Frequency Change and the Estimate