DIGITAL SIGNAL PROCESSING ALGORITHMS FOR POWER AND LINE PARAMETER MEASUREMENTS WITH LOW SENSITIVITY TO FREQUENCY CHANGE

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Abstract - This paper introduces a new approach to definition of digital signal processing algorithms using bilinear form representation. The new algorithms are used to calculate power and line parameter values based on the current and voltage samples. The bilinear form approach provides a convenient methodology for optimal design of digital signal processing algorithms. This feature is utilized to design digital algorithms for power and line parameter measurements with low sensitivity to system frequency change. Several different algorithms are defined and their performance is investigated by testing their sensitivity to system frequency change. Various sampling rates and different data windows are utilized to define several test cases.

Keywords: Power system measurements, Digital signal processing, Power measurements, Line parameter measurements, Bilinear form representation.

INTRODUCTION

A number of different data acquisition, control and protection applications in Electric Power Systems are based on measurements of power and line parameters. Typical examples of power measurements performed for various data acquisition and control applications are active power (P), reactive power (Q), apparent power (S), and power factor (PF). An example of the line parameter measurement associated with the protective relaying applications is the transmission line parameter measurement at the moment of the fault.

Even though the problem of the power and line parameter measurement is well known and understood there are still some new approaches being proposed and implemented. The most recent developments are associated with digital signal processing techniques implemented using microprocessor-based devices. Several different techniques and implementation approaches were proposed for power [1, 2, 3] and line parameter measurements [4, 5, 6].

The new approaches are convenient because of the flexibility and computational capability of the microprocessor-based solutions. However, the main advantage is expected in the area of cost/performance improvements. Examples of the low cost, high performance revenue metering [7] and protective relaying [8] devices can be used as an illustration.

One specific area of concern in the mentioned applications is related to the measurement sensitivity to power system frequency change. Even though the permitted system frequency changes are very small, it is well known that some of the measurements experience significant error due to the frequency change. These errors are undesirable and an efficient method to eliminate and/or to correct the mentioned errors is required [9,10].

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This paper defines a new approach to power and line parameter measurements based on the bilinear form representation of voltages and currents [11]. The advantage of the microprocessor- based technology is utilized by proposing digital signal processing technique for the mentioned measurements. The main benefit of the new approach is obtained by providing an efficient synthesis methodology to develope digital measurement algorithms with low sensitivity to the system frequency change.

The first part of the paper introduces the bilinear form approach used to define digital signal processing algorithms for the mentioned measurements. The following section gives general properties of the bilinear form representation of voltage and current signals. The next section deals with the analysis and synthesis methodology used to define algorithms with low sensitivity to system frequency change. The last section gives test results for some of the algorithms proposed for revenue metering and protective relaying applications.

BILINEAR FORM APPROACH

Active and Reactive Power Calculation

The average power over an interval T is defined as:

$$P(t) = \frac{1}{T}\int_{t-T}^{t} u(t)i(t)dt \tag{1}$$

where u(t) and i(t) designate the voltage and current signal respectively. Several digital algorithms can be defined for calculation of the average power given by equation (1). These algorithms can be divided in two classes, based on the assumptions related to the u(t) and i(t) signals.

The class I algorithms are based on an assumption that the u(t) and i(t) are pure fundamental frequency sinusoids of the following form:

$$\begin{aligned} u(t) &= U_1 sin(\omega t + \varphi_1) \\ i(t) &= I_1 sin(\omega t + \theta_1) \end{aligned} \tag{2}$$

One algorithm can be developed by representing the voltage and current signals by their orthogonal components given by the following discrete-time expressions[2]:

$$\begin{aligned} U_d(n) &= \frac{1}{N}\sum_{k=0}^{N-1} u_{n-k} sin\frac{2\pi}{N}k, \qquad I_d(n) = \frac{1}{N}\sum_{k=0}^{N-1} i_{n-k} sin\frac{2\pi}{N}k \\ U_q(n) &= \frac{1}{N}\sum_{k=0}^{N-1} u_{n-k} cos\frac{2\pi}{N}k, \qquad I_q(n) = \frac{1}{N}\sum_{k=0}^{N-1} i_{n-k} cos\frac{2\pi}{N}k \end{aligned} \tag{3}$$

where the signals are uniformly sampled at frequency $\omega_s = N\omega$. The algorithm for average power calculation can be now expressed, based on the equations (1) and (3), in the following form:

$$\begin{aligned} P &= 2(U_dI_d + U_qI_q) = \\ &= \frac{2}{N^2}\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} u_{n-k} i_{n-m} cos\frac{2\pi}{N}(k-m) \end{aligned} \tag{4}$$

Another class I algorithm can be developed by representing the voltage and current signals, given by equation (2), using the orthogonal set of Walsh functions [3]. The discrete form expressions for voltage and current signals are then given as follows:

$$S_U(n) = \sum_{k=0}^{N-1} u_{n-k} SAL(k/N) \qquad S_I(n) = \sum_{K=0}^{N-1} i_{n-k} SAL(k/N) \tag{5}$$

$$C_U(n) = \sum_{k=0}^{N-1} u_{n-k} CAL(k/N) \qquad C_I(n) = \sum_{K=0}^{N-1} i_{n-k} CAL(k/N)$$

where the signals are uniformly sampled at frequency $\omega_s = N\omega$.

The algorithm for average power calculation for this case can be expressed as follows:

$$P = \frac{(sin\pi/N)^2}{8} \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} u_{n-k} i_{n-m} [SAL(k/N)SAL(m/N)+$$

$$+CAL(k/N)CAL(m/N)] \tag{6}$$

The class II algorithms are based on the assumption that the voltage and current signals are not pure sinusoids, but they are periodical and can be represented as the sum of their Fourier components[1]:

$$u(t) = \sum_{k=0}^{M} U_k sin(k\omega t + \varphi_k)$$

$$i(t) = \sum_{k=0}^{M} I_k sin(k\omega t + \theta_k) \tag{7}$$

The voltage and current signals can be now expressed in discrete-time form as follows:

$$u(n) = \sum_{k=0}^{M} U_k sin(\frac{2\pi}{N} kn + \varphi_k)$$

$$i(n) = \sum_{k=0}^{M} I_k sin(\frac{2\pi}{N} kn + \theta_k) \tag{8}$$

where the signals are uniformly sampled at frequency $\omega_s = N\omega$, $N = 2(M+1)$.

The algorithms for average power can now be given, based on equations (1) and (7), in the following form:

$$P(n) = \sum_{k=0}^{N-1} \frac{1}{N} i_{n-k} u_{n-k} \tag{9}$$

As a conclusion, it can be observed that all of the indicated algorithms, given by equations (4), (6) and (9), can be represented by the following general form:

$$P(n) = \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} u_{n-k} i_{n-m} \tag{10}$$

where h_{km} is a weight attached to the product of voltages and currents.

The expression (10) is designated as a bilinear form of samples of voltage and current signals. A matrix representation of this form, for the power measurement algorithms, can be defined as follows:

$$P = U^T \mathbf{A} I, \qquad Q = U^T \mathbf{B} I \tag{11}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the weight matrices, U and I are the column vectors of $u_n$and i_n respectively, and Q is reactive power.

Line Parameter Calculation

The specific situation considered now is related to the line parameter estimation at the moment of the fault on the line. The transimmission line model considered is the following lumped parameter model:

$$u(t) = Ri(t) + L\frac{di(t)}{d(t)} \tag{12}$$

The line parameters R and L are estimated based on the samples of voltage and currents The parameters are eventually used to determine the fault impedance, which can be expressed by the following relations:

$$Z = R + j\omega L, \quad or \quad Z = \frac{U}{I} \tag{13}$$

In this case, two classes of algorithms for the line parameter calculation can be recognized [8].

The class I algorithms are based on a discrete-time representation of the equation (12):

$$J^1 u_k = RJ^2 i_k + LJ^3 i_k \tag{14}$$

where J^1, J^2, J^3 are the operators used to convert the differential equation (12) into a difference equation. Since equation (14) is an algebraic equation with respect to R and L, two such equations defined for two different time instances are used to obtain estimates of the line parameter:

$$\tilde{R} = \frac{J^1 u_k J^3 i_{k-1} - J^1 u_{k-1} J^3 i_k}{J^2 i_k J^3 i_{k-1} - J^2 i_{k-1} J^3 i_k}$$

$$w\tilde{L} = \frac{J^2 i_k J^1 u_{k-1} - J^2 i_{k-1} J^1 u_k}{J^2 i_k J^3 i_{k-1} - J^2 i_{k-1} J^3 i_k} \tag{15}$$

The class II algorithms are based on the voltage and current signal models given by the following general form:

$$x(t) = x_R cos\omega_o t + x_I sin\omega_o t + \sum k_i \varphi_i(t) \tag{16}$$

where: x_R, x_I –real and imaginary part of the fundamental frequency phasor.

k_i, φ_i – other harmonics and transients

In order to obtain line parameters by estimating the signal components, several algorithms are defined in this class. For an assumption that the fundamental harmonic is the relevant signal model, it is possible to obtain discrete-time estimates of the voltage and current phasor components:

$$\tilde{x}_R = \alpha x_k \qquad \tilde{x}_I = \beta x_k \tag{17}$$

where α and β are operators. As a result, the line parameters can be determined using the estimated values of the phasor components:

$$\tilde{R} = \frac{\alpha u_k \alpha i_k + \beta i_k \beta u_k}{(\alpha i_k)^2 + (\beta i_k)^2}$$

$$\omega\tilde{L} = \frac{\beta u_k \alpha i_k - \alpha u_k \beta i_k}{(\alpha i_k)^2 + (\beta i_k)^2} \tag{18}$$

Finally, all of the mentioned line parameter algorithms, represented by equations (15) and (18), can be represented using the bilinear form approach:

$$R = \frac{U^T \mathbf{C} I}{I^T \mathbf{E} I} \qquad \omega L = \frac{U^T \mathbf{D} I}{I^T \mathbf{E} I} \tag{19}$$

where $\mathbf{C}$, $\mathbf{D}$, and $\mathbf{E}$ are the weight matrices.

As a conclusion, the bilinear form approach is general enough to be used to represent all of the mentioned digital algorithms for power and line parameter measurements. Furthermore, it will be shown that this approach can be used to define a number of new algorithms. This approach also enables development of algorithm analysis and synthesis methodology. Further discussion illustrates how this approach is used to define a specific class of new algorithms for power and line parameter measurements with low sensitivity to system frequency change.

BILINEAR FORM REPRESENTATION PROPERTIES

General Bilinear Form Definition

The general bilinear form of two sequences of samples x_n and y_u is defined by the following expression:

$$BF_n = \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} x_{n-k} y_{n-m} \tag{20}$$

where n is the discrete time when the Bilinear Form value is determined. The term h_{km} is a weight attached to the product of two samples x_{n-k} and y_{n-m}. The bilinear form given by equation (20) is therefore defined by the following weight matrix:

$$\mathbf{H} \hat{=} \{h_{km}\} \tag{21}$$

The matrix dimension is $N \times N$ for the window having the width equal to $(N-1)\Delta t$.

Bilinear Form Value for Harmonic Signals

Let us assume that the fundamental harmonic voltage and current signals are defined as

$$\begin{aligned} u_n &= U\cos(n\psi + \phi) \\ i_n &= I\cos n\psi \end{aligned} \tag{22}$$

where: U, I - signal magnitudes
ϕ - phase between two samples
$\psi = \frac{2\pi\omega_o}{\omega_s}$ - electrical angle between two samples
ω_o - system fundamental frequency
ω_s - sampling frequency

The bilinear form value for the harmonic signals, at the moment $\underline{n}$, defined by equation (22), is given as:

$$\begin{aligned} BF_n &= \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} u_{n-k} i_{n-m} \\ &= U^T \mathbf{H} I \end{aligned} \tag{23}$$

Further expansion of the equation (23) indicates that the bilinear form may be expressed as a sum of a constant term BF^c and a variable term BF_n^v :

$$BF_n = BF^c + BF_n^v \tag{24}$$

These two terms can be represented as a function of the weight matrix $\mathbf{H}$:

$$BF^c = \frac{UI}{2}|H^c(e^{-j\psi})| \cdot \cos\left[\phi + \angle H^c(e^{-j\psi})\right] \tag{25}$$

$$BF_n^v = \frac{UI}{2}|H^v(e^{-j\psi})| \cdot \cos\left[2n\psi + \phi + \angle H^v(e^{-j\psi})\right] \tag{26}$$

where the related weight matrix polinomials H^c and H^v are represented as:

$$H^c(p) = \sum_{r=-N+1}^{+N-1} h_r^c \cdot p^r \tag{27}$$

with:

$$h_r^c = \sum_k \sum_m h_{km} \qquad 0 \le k \le N-1 \tag{28}$$
$$k - m = r \qquad 0 \le m \le N-1$$

and:

$$H^v(p) = \sum_{r=0}^{2N-2} h_r^v \cdot p^r \tag{29}$$

with:

$$h_r^v = \sum_k \sum_m h_{km} \qquad 0 \le k \le N-1 \tag{30}$$
$$k + m = r \qquad 0 \le m \le N-1$$

Analysis of the equations (24), (25) and (26) indicates that the constant part BF^c can be used to determine power and line parameter values. Therefore, it is desirable to define conditions that will make the variable part BF_n^v to be equal zero. The variable term will vanish if the following condition is fulfilled:

$$H^v(e^{-j\psi}) = 0 \tag{31}$$

This is satisfied when $e^{-j\psi}$ is a zero of the polynomial $H^v(p)$. Furthermore, the variable term will vanish for any ψ if $H^v(p)$ is indentically equal to zero. This is the case when:

$$h_r^v = 0 \qquad r = 0, 1, \cdots, 2N-2 \tag{32}$$

Matrix Conditions for Different Measurements

The first set of conditions is related to the selection of matrix $\mathbf{H}$ so that the requirement (31) is satisfied. As indicated by the expression (32), this means that the sums of the matrix elements in the anti-diagonal and all the sub-anti-diagonals have to be zero. Such matrices will be named constant-valued.

The next set of conditions is related to selection of the polynomial $H^c(e^{-j\psi})$ so that the bilinear form value gives active and reactive power.

The active power calculation requires that the $H^c(e^{-j\psi})$ is real for any value of the electrical angle, and equal to 1 for a given value of the angle. Therefore, if the following condition is satisfied:

$$H^c(e^{-j\psi}) = 1, \quad \psi = \psi_0 \tag{33}$$

then it can be seen from equation (25) that:

$$BF^c = \frac{UI}{2}\cos\phi = P \tag{34}$$

It can be easily shown that the symmetric matrices defined as

$$\mathbf{A} = \mathbf{A}^\mathbf{T} \tag{35}$$

satisfy the requirement that their value is always real, i.e, that their imaginary part is always equal to zero:

$$Im\{A^c(e^{-j\psi})\} = 0, \qquad \forall\, \psi \tag{36}$$

However, the symmetric matrices are not necessarily constant-valued, that is also the required condition as expressed by equation (31). One way to construct a constant-valued symmetric matrix A is to choose its elements to satisfy the following conditions:

$$\sum_k \sum_m a_{km} = -\frac{a_{rr}}{2} \qquad r = 0, 1, 2, \cdots N-1 \tag{37}$$

$$0 \le k \le N-1, \;\; 0 \le m \le N-1 \qquad k > m, k+m = 2r$$

$$\sum_k \sum_m a_{km} = 0 \qquad r = 0, 1, 2, \cdots N-1 \tag{38}$$

$$0 \le k \le N-1, 0 \le m \le N-1 \qquad k > m, k+m = 2r+1$$

If the following condition is also satisfied:

$$Re\{A^c(e^{-j\psi})\} \ne 0, \qquad \psi = \psi_0 \tag{39}$$

then a weight matrix for real power calculation can be constructed as

$$\mathbf{H}_p = \frac{1}{Re\{A^c(e^{-j\psi_o})\}} \cdot \mathbf{A} \tag{40}$$

The reactive power calculation requires that the polynomial $H^c(e^{-j\psi})$ is imaginary for any value of the electrical angle, and equal to $-j$ for a given value of the angle. Therefore, the following condition needs to be met:

$$H^c(e^{-j\psi}) = -j, \qquad \psi = \psi_0 \tag{41}$$

In this case the equation (25) gives the value of the reactive power:

$$BF^c = \frac{UI}{2}\sin\phi = Q$$

It can be shown that the skew-symmetric matrices defined as:

$$\mathbf{B}^\mathbf{T} = -\mathbf{B} \tag{42}$$

satisfy the requirement that their value is always imaginary, i.e, that their real part is always equal to zero:

$$Re\{B^c(e^{-j\psi})\} = 0, \qquad \forall\, \psi \tag{43}$$

However, these matrices always satisfy the following condition as well:

$$B^v(e^{-j\psi}) = 0, \qquad \forall\, \psi \tag{44}$$

which is needed to make the variable part of the bilinear form to be equal to zero. In addition, if the following condition is satisfied:

$$Im\{B^c(e^{-j\psi})\} \neq 0, \qquad \psi = \psi_0 \tag{45}$$

then a weight matrix for reactive power calculation can be constructed as:

$$\mathbf{H}_Q = \frac{1}{Im\{B^c(e^{-j\psi_0})\}} \cdot \mathbf{B} \tag{46}$$

The transmission line parameter calculations are based on active and reactive power calculations. Using the following expressions:

$$R = \frac{U}{I}\cos\phi = \frac{\frac{UI}{2}\cos\phi}{\frac{I^2}{2}} = \frac{P}{(RMSI)^2} \tag{47}$$

$$\omega_0 L = \frac{U}{I}\sin\phi = \frac{\frac{UI}{2}sin\phi}{\frac{I^2}{2}} = \frac{Q}{(RMSI)^2} \tag{48}$$

it is possible to determine the required bilinear form matrices from the related power measurement weight matrices. The only additional value that needs to be defined is the RMSI value. The condition for RMSI calculation can be obtained from expression (25) by taking into account that both signals in the bilinear form are the current signals and they are equal with no angle difference between them:

$$BF^c = \frac{I^2}{2} Re\{H^c(e^{-j\psi})\} \tag{49}$$

This brings the new condition:

$$Re\left\{H^c(e^{-j\psi})\right\} = 1 \tag{50}$$

which is needed to make equation (49) to represent the RMSI value. This value is denoted as the quadratic form (QF):

$$QF^c = I^2_{rms} \tag{51}$$

ALGORITHMS WITH LOW SENSITIVITY TO FREQUENCY CHANGE

Power Measurements

The frequency change causes the change in the values of expressions (27) and (29), so that the required conditions given by equations (31), (33) and (41) are not satisfied. This implies that it is desirable that polynomials H^v and H^c show low sensitivity to frequency change. In order to derive the required conditions, let us represent the bilinear form expression, given by equation (24), using the Taylor series expansion. This expansion is performed around the point ψ_0, which is the desired frequency out of the range of the frequency values ψ.

$$BF_n(\psi) = BF_n(\psi_o) + \frac{dBF_n(\psi)}{d\psi}\Big|_{\psi=\psi_o}\frac{\psi-\psi_o}{1!} + \frac{d^2BF_n(\psi)}{d\psi^2}\Big|_{\psi=\psi_o}\frac{(\psi-\psi_o)^2}{2!} + \cdots \tag{52}$$

If only the first two terms are considered, then the low sensitivity frequency change translates into the condition that:

$$\frac{dBF_n(\psi)}{d\psi} = 0 \qquad \text{for } \psi = \psi_o \tag{53}$$

This further means that the following conditions are also satisfied:

$$\frac{dH^v(e^{-j\psi})}{d\psi} = 0 \qquad \text{for } \psi = \psi_o \tag{54}$$

$$\frac{dH^c(e^{-j\psi})}{d\psi} = 0 \qquad \text{for } \psi = \psi_o \tag{55}$$

If the weight matrix $\mathbf{H}$ is selected to satisfy condition (32), then the requirement (54) is satisfied. As far as the polynomial H^c is concerned, both real and imaginary parts have to satisfy the condition (55):

$$\frac{dRe\{H^c(e^{-j\psi})\}}{d\psi} = 0 \qquad \text{for } \psi = \psi_0 \tag{56}$$

$$\frac{dIm\{H^c(e^{-j\psi})\}}{d\psi} = 0 \qquad \text{for } \psi = \psi_0 \tag{57}$$

If the active power calculation is considered, based on the fact that the weight matrices should satisfy the conditions (35) and (36), the imaginary part of the polynomial H^c is indentically equal to zero. Therefore, the condition (56) is the only additional condition required to obtain the low sensitivity to frequency change characteristic.

If the reactive power calculation is considered, based on the conditions (43) and (44), the real part of the polynomial H^c is identically equal to zero. Therefore, the condition (57) is the only additional condition required to obtain the low sensitivity to the frequency change characteristic.

Line Parameter Measurements

Conditions for the calculation of parameter R can be derived by using equations (19), (31), (36) and (39). As a result, the following weight matrix conditions for the estimated value $\tilde{R}$ are obtained:

$$\tilde{R} = \frac{U^T\mathbf{C}I}{I^T\mathbf{E}I} \tag{58}$$

where:

$$C^v(e^{-j\psi}) = 0 \qquad \text{for } \forall\, \psi$$
$$C^c(e^{-j\psi}) = \text{real number} \tag{59}$$
$$E^v(e^{-j\psi}) = 0 \qquad \text{for } \forall\, \psi$$
$$E^c(e^{-j\psi}) = \text{real number} \tag{60}$$

Observing the condition (40) and (49), it is possible to write the expression (58) in the following form:

$$\tilde{R} = \frac{Re\left\{C^c(e^{-j\psi})\right\}\cdot P}{Re\left\{E^c(e^{-j\psi})\right\}\cdot\frac{I^2}{2}} = \frac{Re\left\{C^c(e^{-j\psi})\right\}\cdot P}{Re\left\{E^c(e^{-j\psi})\right\}\cdot I^2_{rms}} \tag{61}$$

Taking into account equation (47), the following relation is obtained for equation (61):

$$\tilde{R} = \frac{Re\left\{C^c(e^{-j\psi})\right\}}{Re\left\{E^c(e^{-j\psi})\right\}}\cdot R \tag{62}$$

Analysis of the equation (62) suggests that the estimated value $\tilde{R}$ is equal to the actual value R, for any value of the angle ψ, if polynomials C^c and E^c are equal:

$$C^c(e^{-j\psi}) = E^c(e^{-j\psi}), \qquad \text{for } \forall\, \psi \tag{63}$$

This translates to the following condition for the corresponding weight matrices:

$$\mathbf{C} = \mathbf{E} \tag{64}$$

Therefore, the condition (64) needs to be satisfied in order to define an algorithm for calculation of the line parameter R so that this algorithm is insensitive to the frequency change. Obviously, in this case the condition (56) is also satisfied.

The conditions for the calculation of the parameter L can be derived in the similar manner by observing equations (19), (31), (43) and (45). In this case the estimated value $\tilde{L}$ can be obtained as:

$$\tilde{L}\omega_0 = \frac{U^T\mathbf{D}I}{I^T\mathbf{E}I} \tag{65}$$

under the following conditions

$$D^v(e^{-j\psi}) = 0 \qquad \text{for } \forall\, \psi$$
$$D^c(e^{-j\psi}) = \text{imaginary number} \tag{66}$$

$$E^v(e^{-j\psi}) = 0 \qquad \text{for } \forall\, \psi$$
$$E^c(e^{-j\psi}) = \text{real number} \tag{67}$$

Further derivation is based on the equations (46) and (49). Using these equations it is possible to write the equation (65) as:

$$\begin{aligned} \tilde{L} &= -\frac{1}{\omega_0}\frac{Im\{D^c(e^{-j\psi}\}}{Re\{E^c(e^{-j\psi}\}} \cdot \frac{Q}{\frac{I^2}{2}} \\ &= -\frac{1}{\omega_0}\frac{Im\{D^c(e^{-j\psi})\}}{Re\{E^c(e^{-j\psi})\}} \cdot \frac{Q}{I^2_{rms}} \end{aligned} \tag{68}$$

Taking into account equation (48), the following relation is obtained for equation (68):

$$\tilde{L} = -\frac{1}{\omega_0}\frac{Im\{D^c(e^{-j\psi})\}}{Re\{E^c(e^{-j\psi})\}} \cdot L\omega \tag{69}$$

Further rearrangements of equation (69) are needed:

$$\tilde{L} = -\frac{Im\{D^c(e^{-j\psi})\}}{Re\{E^c(e^{-j\psi)}\}} \cdot \frac{L\omega}{\omega_0} \cdot \frac{\Delta t}{\Delta t} \tag{70}$$

to obtain the final expression as:

$$\tilde{L} = -\frac{Im\{D^c(e^{-j\psi})\}}{Re\{E^c(e^{-j\psi})\}} \cdot \frac{\psi}{\psi_0} \cdot L \tag{71}$$

In order to make the estimated value $\tilde{L}$ to be equal to the actual value L, it is needed that:

$$-Im\left\{D^c(e^{-j\psi})\right\}\psi = Re\left\{E^c(e^{-j\psi})\right\}\psi_0 \tag{72}$$

If also the conditions (56) and (57) are needed in order to provide the low sensitivity to the frequency change, then the new condition for the equation (72) is:

$$-\frac{d}{d\psi}Im\left\{D^c(e^{-j\psi})\psi\right\}\bigg|_{\psi=\psi_0} = \frac{d}{d\psi}Re\left\{E^c(e^{-j\psi})\psi_0\right\}\bigg|_{\psi=\psi_0} \tag{73}$$

ALGORITHM TESTING

Power Measurements

Algorithm P1 - This algorithm is derived for active power calculations for 2-sample data window. As it is shown in the Appendix I, the weight matrix is given as:

$$\mathbf{H}_p = \frac{1}{2\sin^2\psi_0}\begin{bmatrix} 1 & -\cos\psi_0 \\ -\cos\psi_0 & 1 \end{bmatrix} \tag{74}$$

If we take the frequency of $f_0 = 60Hz$, since $\psi_o = 60°$ the polynomials H^c and H^v are respectively equal to:

$$H^c(e^{-j\psi_0}) = -\frac{2}{3}\cos\psi_0 + \frac{4}{3} = 1 \tag{75}$$

$$H^v(e^{-j\psi_0}) = 0 \tag{76}$$

The equations (75) and (76) represent conditions for active power measurement. However, if the frequency should change, then the value of both polynomials changes and the value of the calculated power changes as a consequence. The percentage of the power change, for any given frequency change is given in Table I.

Table I. Active Power Errors for 2-Sample Algorithm

$f\%$	-10.000	-5.000	-2.000	f_0	$+2.000$	$+5.000$	$+10.000$
H^c	0.942	0.970	0.987	$H^c = 1$	1.012	1.031	1.062
P%	-6.000	-3.000	-1.000	0	$+1.000$	$+3.000$	$+6.000$

Algorithm P2 - This algorithm is derived for active power calculation for 3-sample data window. Derivation similiar to the one give for algorithm P1 will produce the following weight matrix:

$$\mathbf{H}_p = \frac{1}{4\sin^2\psi}\begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{77}$$

In this case the sum of anti-diagonal elements is equal to zero which means that the variable term $BF^v_n = 0$ for any ψ. The constant term is given for $\psi_o = 60°$ as:

$$H^c(e^{-j\psi}) = \frac{4}{3}\sin^2\psi \tag{78}$$

The value of the constant term for frequency of $f = 60Hz$ is equal to one, that is the condition for active power measurement. However, as the frequency changes, the constant term will change and the calculated active power will change as a consequence. The results of this change are given in Table II. Analysis of the errors at reference frequency is given for different sampling rates. Errors for power calculations P^1_2, P^2_2 and P^3_2 correspond to the electrical angle between samples of $\psi = 60°, 45°$ and $10°$ respectively.

Table II. Active Power Errors for 3-Sample Algorithm

$f\%$	-5.00	-2.50	f_0	$+2.50$	$+5.00$
$P^1_2\%$	-6.30	-3.20	0	$+2.90$	$+5.80$
$f\%$	-5.00	-2.00	f_0	$+2.00$	$+5.00$
$P^2_2\%$	-8.80	-3.30	0	$+3.10$	$+8.60$
$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$P^3_2\%$	-18.70	-9.30	0	$+10.50$	$+20.80$

Algorithm Q1 - This algorithm is derived for reactive power calculation for 3-sample data window. Derivation similar to the one given for algorithm P1 will generate the following weight matrix:

$$\mathbf{H}_Q = \frac{1}{2\,\sin 2\psi}\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{79}$$

Again, the variable part of the bilinear form is equal to zero, and the constant part can be presented as(for$\psi = 15°$):

$$H^c(e^{-j\psi}) = -2\text{j}\sin 2\psi \tag{80}$$

The analysis of the expression (80) indicates that this algorithm has a very low sensitivity to frequency change at the reference frequency. The results of the power calculation errors obtained as a result of the sampling rate change are given in Table III. The calculated errors for reactive power Q^1_1 and Q^2_1 correspond to the electrical angle between samples of $\psi = 15°$ and $45°$ respectively.

Table III. Reactive Power Errors for 3-Sample Algorithm

$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$Q^1_2\%$	-9.30	-4.30	0	$+4.20$	$+8.90$
$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$Q^2_2\%$	-1.30	-0.30	0	-0.30	-1.30

Algorithm P3 - This algorithm is derived to satisfy the low sensitivity to frequency change requirement. As it is shown in Appendix II, the weight matrix for 4-sample data window is of the following form:

$$\mathbf{H}_p = \begin{bmatrix} 0 & 0 & a & b \\ 0 & -2a & -b & 0 \\ a & -b & 0 & 0 \\ b & 0 & 0 & 0 \end{bmatrix} \tag{81}$$

where:

$$a = \frac{\cos 2\psi_o + \cos^2\psi_o}{4\sin^4\psi_o}, \quad b = -\frac{\cos\psi_o}{4\sin^4\psi_o} \tag{82}$$

The results for active power calculation errors due to the sampling rate change are given in Table IV. As it can be observed, the results are given for two different sampling rate that produce power calculation errors P^1_3 and P^2_3 for the corresponding electrical angles of $\psi = 60°$ and $30°$ respectively

Table IV. Active Power Errors for 4-Sample Algorithm with Low Sensitivity to Frequency Change.

$f\%$	-5.00	-2.00	f_0	$+5.00$	$+10.00$
$P_3^1\%$	-0.80	-0.15	-0	-0.10	-0.60
$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$P_3^2\%$	-3.10	-0.80	0	-0.70	-3.60

Algorithm P4-This algorithm is also defined to satisfy the low sensitivity to frequency change requirement. However, the difference between this algorithm and algorithm P3 is in the level of the "insensitivity" requirements. This algorithm has the following additional requirement:

$$\frac{d^2H^c}{d\psi^2} = 0 \tag{83}$$

If the condition (83) is satisfied, and if the 5-sample algorithm is selected, then the related weight matrix is given as:

$$\mathbf{H}_p = \begin{bmatrix} 0 & 0 & -a & b & -c \\ 0 & 2a & -b & 0 & 0 \\ -a & -b & 2c & 0 & 0 \\ b & 0 & 0 & 0 & 0 \\ -c & 0 & 0 & 0 & 0 \end{bmatrix} \tag{84}$$

In this case the power calculation errors are reduced even more. This is illustrated in Table V for different sampling rates that correspond to $\psi = 60°, 30°$ and $45°$. The related power error calculations are $P_4^1, P_4^2, and P_4^3$ respectively.

Table V. Active Power Error for 5-Sample Algorithm with Low Sensitivity to Frequency Change.

$f\%$	-5.00	-2.00	f_0	$+2.00$	$+5.00$
$P_4^1\%$	-0.01	$-0.35 \cdot 10^{-4}$	0	$+0.35 \cdot 10^{-2}$	$+0.05$
$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$P_4^2\%$	-0.60	-0.08	0	$+0.09$	$+0.78$
$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$P_4^3\%$	-0.50	-0.10	0	$+0.10$	$+0.60$

Line Parameter Measurements

Algorithm L1 - This algorithm is based on 2-sample data window and has the following weight metrics:

$$\mathbf{D} = \frac{1}{2sin\psi}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{E} = \frac{1}{2sin^2\psi}\begin{bmatrix} 1 & 0 \\ -2cos\psi & 1 \end{bmatrix} \tag{85}$$

However, this algorithm does not satisfy the low sensitivity to frequency change requirement since it produces the variable part E^v. Therefore, the expression for calculation of the parameter $\tilde{L}$, using this algorithm, is equal to:

$$\tilde{L} = \frac{-Im\{D^c(e^{-j\psi})\}}{E^v(e^{-j\psi}) + Re\{E^c(e^{-j\psi})\}} \frac{\psi}{\psi_o} \cdot L \tag{86}$$

This algorithm is quite inaccurate when the frequency changes. If the oscillatory term E^v is taken out, the results obtained for calculation of $\tilde{L}$ are improved. The results for this case are given in Table VI for several values of the sampling rates that correspond to $\psi = 60°$, $45°$ and $30°$.

Table VI. Parameter L Errors for 2-Sample Algorithm.

$f\%$	-10.00	-5.00	-2.00	f_0	$+2.00$	$+5.00$	$+10.00$
$L_1^1\%$	-11.00	-5.30	-2.00	0	$+1.90$	$+4.60$	$+9.10$
$f\%$	-10.00	-5.00	-2.00	f_0	$+2.00$	$+5.00$	$+10.00$
$L_1^2\%$	-10.70	-5.20	-2.10	0	$+1.80$	$+4.60$	$+9.20$
$f\%$	-10.00	-5.00	-2.00	f_0	$+2.00$	$+5.00$	$+10.00$
$L_1^3\%$	-11.20	-5.60	-2.60	0	$+1.60$	$+4.40$	$+8.90$

Algorithm L2 - This algorithm is based on 3-sample data window and has the following weight matrices.

$$\mathbf{D} = \begin{bmatrix} 0 & a & 0 \\ -a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{87}$$

The condition that:

$$Re\{E^c(e^{-j\psi_o})\} = -ImD^c\{(e^{-j\psi_o})\} \tag{88}$$

produces the requirement $a = 2sin\psi_o$ that should be used in expressions (89) in order to calculate the parameter L. The results of the parameter L calculation errors for different sampling rates, are given in Table VII. As it can be observed, the sampling rates of $\psi = 60°$ and $45°$, and $30°$ correspond to parameter values L_2^1, L_2^2, L_2^3 respectively.

Table VII. Parameter L Errors for 3-Sample Algorithm.

$f\%$	-5.00	-2.00	f_0	$+2.00$	$+5.00$
$L_2^1\%$	-1.90	-0.89	0	$+0.68$	$+2.00$
$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$L_2^2\%$	-2.00	-1.20	0	$+1.00$	$+2.20$
$f\%$	-10.00	-5.00	f_0	$+5.00$	$+10.00$
$L_2^3\%$	-0.90	-0.50	0	$+0.40$	$+1.00$

Algorithm L3 - This algorithm is designed for low sensitivity to frequency change. If the 3-sample data window is considered, then the required weight matrices are given as follows:

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 & a & b \\ -a & 0 & 0 \\ -b & 0 & 0 \end{bmatrix} \tag{89}$$

The low sensitivity to frequency change requires that the following conditions are satisfied:

$$\frac{d}{d\psi}[Im\{D^c(c^{-j\psi})\}\psi]\Big|_{\psi=\psi_o} = -\frac{d}{d\psi}[Re\{E^c(e^{-j\psi})\}\psi_o]\Big|_{\psi=\psi_o} \tag{90}$$

in addition to the requirement (88). As a result, the following values for the parameters a and b are derived:

$$a = \frac{-2\psi_o + sin2\psi_o}{\psi_o sin\psi_o}, \quad b = \frac{-sin\psi_o + \psi_o cos\psi_o}{\psi_o sin\psi_o} \tag{91}$$

This set of conditions is derived in the Appendix III. Finally, the error calculations for parameter L are given in Table VII for several sampling rates. The parameter L calculation errors L_3^1, L_3^2, L_3^3, and L_3^4 are shown for sampling rates of $\psi = 60°$, $45°$, $30°$, and $10°$. The results show indeed a very low sensitivity to frequency change.

Table VIII. Parameter L Errors for 3-Sample Algorithm with Low Sensitivity to Frequency Change.

$f\%$	-10.00	-5.00	-2.00	f_0	$+2.00$	$+5.00$	$+10.00$
$L_3^1\%$	-0.09	-0.10	-0.12	0	-0.10	-0.10	-0.30
$f\%$	-10.00	-5.00	-2.00	f_0	$+2.00$	$+5.00$	$+10.00$
$L_3^2\%$	-0.33	-0.60	-0.55	0	-0.40	-0.40	-0.30
$f\%$	-10.00	-5.00	-2.00	f_0	$+2.00$	$+5.00$	$+10.00$
$L_3^3\%$	$+0.18$	$+0.14$	$+0.10$	0	$+0.10$	0.19	$+0.13$
$f\%$	-10.00	-5.00	-2.00	f_0	$+2.00$	$+5.00$	$+10.00$
$L_3^4\%$	$+0.78$	$+0.75$	$+0.77$	0	$+0.80$	$+0.70$	$+0.72$

CONCLUSIONS

Use of the bilinear form representation of current and voltage signals has provided quite convenient and powerful methodology for synthesis of the new class of digital signal processing algorithms. These algorithms can be used for calculation of power and line parameter values related to power system measurements. The major advantages of the new approach can be summarized as follows:

- The new approach is convenient because it defines the synthesis methodology for algorithm design. This methodology can also be used to make a consistent analysis of the existing algorithms which was difficult to perform in the past when only the heuristic definition of these algorithms were given.

- The new approach is powerful because it provides a methodology for optimization of the algorithm performance. In particular, the low sensitivity to the frequency change optimization for the algorithm was demonstrated by designing several algorithms for power and line parameter measurements with a very low sensitivity to frequency change.

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Appendix I

Conditions (31) and (32) translate into:

$$\begin{aligned} a + (c+b)e^{-j\psi} + g \cdot e^{-2j\psi} &= 0 \\ be^{j\psi} + (a+g) + c \cdot e^{-j\psi} &= 1 \end{aligned} \qquad (A-1)$$

Solving this set of equations gives coefficients of equation (74).

Appendix II

The polynomial H^c and its derivative are:

$$H^c = 2bcos3\psi + 2acos2\psi - 2bcos\psi - 2a \qquad (A-2)$$

$$\frac{dH^c}{d\psi} = -6bsin3\psi - 4asin2\psi + 2bsin\psi \qquad (A-3)$$

Solution of these equations yields the values of the parameters a and b as they are given by equations (82).

Appendix III

The related polynomials are as follows:

$$Re\{E^c\} = 4sin^2\psi, \quad Im\{D^c\} = 2asin\psi + 2bsin2\psi \quad (A-4)$$

The condition (72) gives:

$$a = -2sin\psi_o - 2bcos\psi_o \qquad (A-5)$$

The condition (90) gives:

$$b = \frac{-sin\psi_o + \psi_o cos\psi_o}{\psi_o sin\psi_o} \qquad (A-6)$$

Solution of equations (A-5) and (A-6) gives parameters given in equation (91).

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