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# DIGITAL PROTECTIVE RELAYING ALGORITHM SENSITIVITY STUDY AND EVALUATION

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Abstract - This paper gives the results of an experimental study undertaken to analyze parameter sensivity of the digital protective relaying algorithms and to develop a methodology for evaluating those algorithms. The study is based on computer simulations of relaying environments and the algorithms. Representative algorithms and test conditions were chosen based on a theoretical study of previously published algorithms. An algorithm sensitivity study to various power system conditions and algorithm parameters is presented and the algorithms are evaluated using an adaptive criterion function developed in the study.

## INTRODUCTION

Several studies have been published that have compared and evaluated digital algorithms for transmission line protective relaying [1,2,3,4]. These studies ranged from analyses to identify common properties in some algorithms to algorithm testing based on computer simulated and/or recorded fault transients.

This paper presents the results of an experimental study which was supported by and supplemented by theoretical analysis [5]. This work classified published algorithms and formed the basis for selecting representative algorithms, identifying test conditions, and, in several instances, explaining the test results. The purposes of the experimental study were to analyze the sensitivity of the algorithms to variations in power system canditions and algorithm parameters; to define a criterion function to judge algorithm performance for particular relaying conditions; and to evaluate representative algorithms for several sets of protective relaying application conditions. In summary, the paper presents a consistent methodology which can be used to compare and evaluate digital protective relaying algorithms.

## EVALUATION STUDY STRUCTURE AND PARAMETERS

The structure for evaluating algorithm performance is shown graphically in Figure 1. The Bonneville Power Administration Electromagnetic Transient Program (EMTP) was used to produce simulated fault transients for a variety of system configurations and conditions. To obtain valid simulation results a small integration time step was required. A software package was developed to decimate the data to the required effective sampling rates for algorithm evaluation. This package was designed so that any analog filter described via a transfer function can be accurately simulated and used as the low pass filter required for decimation. A second software package was developed to implement the algorithms. This package produced the estimated values of R, X, and Z at each time step and the post fault means and standard deviations.

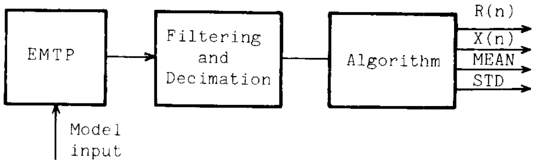

Figure 1. Evaluation Structure

The evaluation structure enables the algorithm performance to be tested and judged for a variety of power system test conditions and a range of algorithm or relay implementation conditions.

The first set of test conditions consists of power system related conditions, i.e. conditions related to particular protective relaying environments. These conditions affect the signal content at the moment of the fault. System conditions and/or parameters varied for testing were:

- "short" transmission line
- "long" transmission line
- series compensated line
- parallel compensated line
- system frequency
- fault location
- type of the fault
- fault angle
- fault resistance

To enable this range of conditions to be tested two base EMTP models were generated. Model 1 is a 27.93 mile 138 kV line and Model 2 is a 122 mile 138 kV line. The Semlyen modeling option of EMTP was used. Appendix I and II contain diagrams of the systems configuration for both models, the pertinent system parameters used in the models, and the test fault conditions for which transients were generated.

The second set of test conditions are for the design parameters of a digital relay. They affect the analog signal processing and the algorithm parameters. These conditions include:

- sampling rate
- low-pass filter cut-off frequency
- data window width
- digital relay response time

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The sampling rates selected for testing were 96, 32, 24, 16, 12, 8 and 4 s/c. This choice was based on the suggested sampling rates found in the literature as well as constraints imposed by the microprocessor technology which is presently available for digital relay implementation.

Two low-pass filters were selected for the study: one third order Butterworth filter with a cut-off frequency of 320 Hz and one first order filter with a cut-off frequency of 800 Hz. These filters were selected after initial testing with seven first order filters with cut-off frequencies ranging from 110 Hz to 2640.

Some algorithms are amenable to data window variation while some, due to their inherent derivation assumptions, are not. Algorithm data windows were varied in the algorithms for which the theoretical analysis indicated it was appropriate. In these cases data windows of length 0.25T, 0.5T and T were selected.

Digital relay response time is the time interval after a fault which is sufficient for a trip/no trip decision to be made. Three time intervals were considered in the study: (0.25-0.75)T, (0.5-1.5)T and (1.5-2.5)T where T is one cycle. These time intervals were chosen to correspond to a fast relay, a medium speed relay and a relatively slow relay.

In the study over 10,000 simulation runs were made to investigate all combinations of these test conditions. The results given in the paper are selected examples of the most interesting cases.

## SELECTION OF THE ALGORITHMS

The algorithms selected for the simulation study are summarized in Table I. The algorithms were selected based on the theoretical study which classified the previously published algorithms [5].

A total of fourteen algorithms were selected. Five algorithms, denoted C1-1 to C1-5 are Class I algorithms which are based on a differential equation model of a transmission line. The nine algorithms denoted C2-1 to C2-9 are Class II algorithms which are based on input-output models of the post fault signal.

Algorithm C1-1, which uses a "backward" derivative approximation, was chosen to represent the subclass of Class I in which the derivative term in the model is directly approximated. Algorithms C1-2 to C1-5 were selected to represent the variety of ways the integration approach has been applied to the differential equation model.

Class II contains two major subclasses. The first subclass consists of those algorithms in which no optimization is applied to identify parameters in the signal model. Algorithm C2-1 to C2-4 represent a further subclass where only the fundamental frequency component is modeled. Algorithm C2-5 and C2-6 represent the group of algorithms in which other components of the signal are modeled as well.

The second subclass of Class II consists of algorithms which are based on various optimization techniques. Algorithms C2-7 and C2-8 represent the subclass of nonrecursive approaches and C2-9 represents the subclass of recursive approaches.

Not all of the lower levels of the algorithm subclasses identified in the theoretical study are enumerated here and some algorithms which have been reported in the literature were tested but not reported here for space limitations. However, the selected set of algorithms is believed to be sufficient to illustrate all of the major properties of the most significant algorithm subclasses.

Table I. Relaying Algorithms Selected

| Class and No. | Algorithm | Ref. | Mathematical forms |
|---|---|---|---|
| C1-1 | Bornard, Bastide | 6 | $u_1+e_1=Ri_1+L\frac{i_1-i_0}{\Delta T}$ ; data window (DW) DW: T/4 |
| C1-2 | Mc Innes, Morrison | 7 | $\int_{t_1}^{t_2} u\,dt=\int_{t_1}^{t_2} Ri\,dt+\int_{t_1}^{t_2}\frac{di}{dt}\,dt;\ \int_{t_3}^{t_4} u\,dt=\int_{t_3}^{t_4} Ri\,dt+\int_{t_3}^{t_4}\frac{di}{dt}\,dt;\ t_2-t_1=t_4-t_3=T/4$ DW: T/2 |
| C1-3 | Mc Innes, Morrison | 7 | as in C1-2 $t_2-t_1=t_4-t_3=T/8$ DW: T/4 |
| C1-4 | Breingan et al. | 8 | $\frac{u_1+u_0}{2}=R\frac{i_1+i_0}{2}+L\frac{i_1-i_0}{\Delta T}$, $\frac{u_2+u_1}{2}=R\frac{i_2+i_1}{2}+L\frac{i_2-i_1}{\Delta T}$ |
| C1-5 | Ranjbar, Cory | 9 | $\int_{t_1}^{t_2} u\,dt+\int_{t_3}^{t_4} u\,dt=R\int_{t_1}^{t_2} i\,dt+R\int_{t_3}^{t_4} i\,dt+L\int_{t_1}^{t_2}\frac{di}{dt}\,dt+L\int_{t_3}^{t_4}\frac{di}{dt}\,dt \quad t_2-t_1=t_4-t_3=T/4$<br>$\int_{t_1'}^{t_2'} u\,dt+\int_{t_3'}^{t_4'} u\,dt=R\int_{t_1'}^{t_2'} i\,dt+R\int_{t_3'}^{t_4'} i\,dt+L\int_{t_1'}^{t_2'}\frac{di}{dt}\,dt+L\int_{t_3'}^{t_4'}\frac{di}{dt}\,dt \quad t_3-t_1=T/4,\ t'=t+2\Delta T$ |
| C2-1 | Lobosh | 10 | $R=\frac{u_2(i_4-i_2)-u_3(i_3-i_1)}{i_2(i_4-i_2)-i_3(i_3-i_1)}$ $X=\frac{2(u_3i_2-u_2i_3)\sin\omega\Delta T}{i_2(i_4-i_2)-i_3(i_3-i_1)}$ |
| C2-2 | Gilcrest et al. | 11 | $u_1'=Ri_1'+Li_1''$, $u_1''=Ri_1''-\omega^2Li_1'$, $u_1'=\frac{1}{2\Delta T}(u_2-u_0)$, $u_1''=(\frac{1}{\Delta T})^2(u_2-2u_1+u_0)$ |
| C2-3 | Mann, Morrison | 12 | $u_1=Ri_1+Li_1'$ $u_1'=Ri_1'-\omega^2Li_1$ $u_1'=\frac{1}{2\Delta T}(u_2-u_0)$ |
| C2-4 | Gilbert, Shovlin | 13 | $R=\frac{2u_1i_1-u_2i_0-u_0i_2}{2(i_1^2-i_2i_0)}$ $X=\frac{u_1i_2-u_2i_1}{i_1^2-i_2i_0}\sin\omega\Delta T$ |
| C2-5 | Slemon et al. | 14 | Fourier analysis - Full cycle |
| C2-6 | Hope et al. | 15 | $\Psi_{ui}(\tau)=\frac{1}{N}\sum_{K=1}^{N}u(K\,T/N)\,i(K\,T/N+\tau)$, $R=\Psi_{ui}(0)/\Psi_{ii}(0)$, $X=\frac{\Psi_{ui}(\frac{\pi}{2\omega})}{\Psi_{ii}(\pi/2\omega)}-R$ |
| C2-7 | Brooks | 16 | $X(t)=K1+K2\sin(\omega t+\vartheta)$, $\sin\omega t\cong\omega t-(\omega t)^3/3!$ $\cos\omega t\cong 1-(\omega t)^2/2!$ |
| C2-8 | Thorp et al. | 17 | Fourier analysis-half cycle |
| C2-9 | Girgis | 18 | Kalman filter ing. For the voltage: two states model. For the current: three states model |

## SENSITIVITY STUDY

Extensive tests were conducted in which all of the parameters enumerated above were varied. Due to space limitations the results are presented here only for the parameters that showed signficant sensitivity. The sensitivity of the following parameters is discussed:

-system frequency
-line compensation capacitance
-sampling rate
-low-pass filter cut-off frequency

Appendix III contains a summary of the base test conditions and range of parameter variation for all parameter sensitivity studies presented below. The results of each parameter study are summarized in tables where appropriate to conserve space. Only a representative sample of the best, the worst, or some of the most interesting cases are included. Sample algorithms are always included from both classes. The algorithm numbering scheme corresponds to that in Table I. The measure of sensitivity is described by the following values: MEAN - the time average of the R, X, or Z estimate over the specified interval, and STD -the standard deviation for R, X, or Z estimate over the indicated time interval. (These terms are defined more precisely in the next section.)

Each of the subsections below contains the reasons for the test conditions selected, a commentary and explanation of the overall results, and, where appropriate, summary tables. Again, where pertinent, the theoretical study is used to explain the results.

### System Frequency

Sample results are given in Table II (The first three are good results, the last two poor results). The (1.5-2.5)T time interval was selected because, in this interval, the fundamental harmonic is disturbed the least by other signal components. The other test conditions (summarized in Appendix III) were selected to represent a typical digital protective relaying situation.

Table II. Frequency Sensitivity

| | f=60 Hz | | f=63 Hz | | f=57 Hz | |
|---|---|---|---|---|---|---|
| Alg. | R | X | R | X | R | X |
| C1-3 | 70.13 | 67.52 | 70.57 | 67.40 | 69.75 | 67.68 |
| C2-4 | 69.88 | 68.47 | 70.28 | 68.60 | 69.51 | 68.37 |
| C2-1 | 70.02 | 68.44 | 70.53 | 68.59 | 69.57 | 68.35 |
| C2-6 | 62.14 | 57.93 | 62.95 | 65.74 | 62.94 | 51.43 |
| C2-9 | -5.85 | 72.72 | 227.68 | 70.62 | -224.93 | 65.75 |

The results are expressed in terms of the mean value of R and X. Algorithm C1-3 exhibited the smallest deviation over the range of frequencies tested. Two Class II algorithms with built in frequency compensation, C2-4 and C2-1, also exhibited small deviations. Algorithms C2-6 and C2-9 exhibited the largest changes. These results are consistent with the theoretical analysis and classification [5].

The analysis of the entire set of test runs revealed a group of algorithms for which the impedance varied less than 1% over the frequency range tested if the sampling rate was above a threshold. If the sampling rate is above 8 s/c, algorithms C1-2, C1-3, C1-4, and C1-5 met this criteria. Algorithms C2-1 and C2-4 met the criteria if the sampling rate was above 12 s/c. These are Class II algorithms which have built in frequency compensation. The remaining algorithms in Class II exhibited derivations ranging from a minimum of 4% to very high percentages.

### Line Compensation

To determine algorithm sensitivity to this parameter Model 2 was used as the reference case. Series or parallel compensation capacitors were placed at nodes S1 and S7 of the reference figure in Appendix I. Appendix III indicates the remaining test conditions used. Faults were placed at the 11% and the 68.7% locations. The time intervals were restricted to the first (0.25-0.75)T and third (1.5-2.5)T intervals. Sensitivity was assessed by calculating the percent differences in the mean value of reactance between each compensated case and the corresponding uncompensated case.

Analysis of the complete set of test cases revealed that the algorithms were, in general, less sensitive to serial compensation than parallel compensation. For the series compensated line and using the first interval algorithm C1-4 had the least deviation of 0.1% while algorithm C2-3 had the greatest deviation of 15%. Using the third interval, algorithm C2-1 had a deviation of 0.7% while C2-9 had a deviation of 8%. In all cases fault location did not prove to have much influence.

For the parallel compensated line, compensation had more of an effect for far faults than for near faults. In the first interval, algorithm C2-1 exhibited the smallest variation (1.5%) while algorithm C1-4 varied 63%. In the third interval, algorithms C2-2, C2-4 and C1-4 all had variations greater than 50% while the remaining algorithms showed variations less than 3%.

The study verified expectations that, in general, the algorithms are very sensitive to this condition.

### Sampling Rate

Tables III and IV contain the means and standard deviations of the impedance estimated by algorithms C1-1, C1-4, C2-1, C2-5, and C2-9 for sampling rates over the range from 4 s/c to 96 s/c. These five algorithms were chosen to represent the major subclasses of Class I and Class II. The remaining test conditions are as indicated in Appendix III.

The nominal value of estimated impedance is 61.27%. The mean values in the range between 60.2% and 62.2% are underscored in Table III. In Table IV, the minimum value of standard deviation for each algorithm is also underscored. Note the difficulty in trying to predict an "optimal" sampling rate. Both mean and standard deviations must be analyzed. The most accurate mean and the smallest standard deviation do not always occur at the same sample rate. The appropriate tradeoff must be determined for each algorithm and specific system applications.

Table III. Mean (Z) Sensitivity vs. Sampling Rate

| Alg. | s=96 | s=32 | s=24 | s=16 | s=12 | s=8 | s=4 |
|---|---|---|---|---|---|---|---|
| C1-1 | 60.80 | 60.83 | 61.06 | 62.00 | 63.61 | 69.04 | 113.53 |
| C1-4 | 61.14 | 60.92 | 60.78 | 60.35 | 59.75 | 58.03 | 48.45 |
| C2-1 | 61.06 | 61.05 | 61.05 | 61.05 | 61.04 | 61.03 | 61.02 |
| C2-5 | 61.31 | 61.31 | 61.31 | 61.31 | 61.30 | 61.27 | 61.10 |
| C2-9 | 70.65 | 67.32 | 65.03 | 63.31 | 63.16 | 61.25 | 61.88 |

Table IV. Standard Deviation (Z) Sensitivity vs. Sampling Rate

| Alg. | s=96 | s=32 | s=24 | s=16 | s=12 | s=8 | s=4 |
|---|---|---|---|---|---|---|---|
| C1-1 | 0.63 | 1.08 | 1.69 | 3.06 | 4.63 | 8.59 | 46.17 |
| C1-4 | 2.99 | 0.99 | 0.96 | 0.99 | 1.04 | 1.22 | 3.27 |
| C2-1 | 1.51 | 0.92 | 0.91 | 0.91 | 0.91 | 0.89 | 0.86 |
| C2-5 | 4.42 | 4.42 | 4.42 | 4.43 | 4.44 | 4.49 | 4.77 |
| C2-9 | 1.31 | 1.01 | 0.82 | 0.70 | 0.74 | 0.88 | 1.61 |

## Low-Pass Filter

Tables V and VI contain sample results for a subset of the test cases in which algorithms were tested with a 320 Hz third order Butterworth filter and with no filter. These tables show the means and standard deviations of the impedance estimates for the three best cases (first three) and two worst cases for sampling rates of 16 s/c and 32 s/c and the other conditions indicated in Appendix III.

The analysis of the results of the entire set of tests indicated that some algorithms were not particularly sensitive to the characteristics or presence of a filter. Algorithms with a relatively wide data window (over 0.25T) exhibited this characteristic. Also, it appeared that algorithms with a relatively short data window (less than 0.25T) were sensitive to the characteristics or presence of the filter for high sampling rates (greater than or equal to 32 s/c).

Table V. Filter Cutoff Frequency Sensistivity-16 s/c

| MEAN (Z) | | | STD (Z) | | |
|---|---|---|---|---|---|
| Alg. | No Filter | Filter | Alg. | No Filter | Filter |
| C2-7 | 58.96 | 58.95 | C1-2 | 17.71 | 17.74 |
| C1-3 | 62.81 | 62.94 | C2-8 | 49.63 | 49.77 |
| C1-2 | 62.88 | 62.74 | C1-1 | 19.74 | 20.71 |
| C2-1 | 110.65 | 59.85 | C2-1 | 1249.40 | 20.01 |
| C2-2 | 48.07 | 62.55 | C2-2 | 301.11 | 99.73 |

Table VI. Filter Cutoff Frequency Sensitivity-32 s/c

| MEAN (Z) | | | STD (Z) | | |
|---|---|---|---|---|---|
| Alg. | No Filter | Filter | Alg. | No Filter | Filter |
| C2-1 | 60.79 | 60.78 | C2-7 | 23.76 | 23.79 |
| C1-4 | 64.04 | 63.88 | C2-1 | 23.82 | 24.02 |
| C2-3 | 63.55 | 63.22 | C1-4 | 25.22 | 25.18 |
| C2-2 | 65.85 | 64.42 | C2-2 | 35.26 | 24.90 |
| C2-9 | 58.47 | 57.02 | C2-9 | 26.05 | 22.83 |

## A CRITERION FUNCTION FOR ALGORITHM EVALUATION

All empirical studies to evaluate protective relaying algorithms have faced the task of evaluating and reducing an extreme volume of test data. In this study the combinations of all power system and relaying parameters resulted in over 10,000 test results to be evaluated. The raw algorithm outputs are the estimates of R, X, or Z at each sampling instant, and these must be evaluated with respect to different intended relay applications and with respect to a variety of different trip decision algorithms.

### Criterion Function Definition

A criterion function has been developed which enables algorithms to be quantitatively ranked for these sets of conditions. This function is derived and interpreted in the following.

Let the actual value of the parameter used to make a tripping decision be denoted by Z(t). This parameter changes from a prefault value of $Z_{pr}$ to a post fault value of $Z_{po}$. Let the calculated or estimated values of this parameter in discrete time n be Z(n). Their relation is defined by,

$$Z(n) = Z_{po} + S(n) \tag{1}$$

where S(n) is the error of the estimate. A common technique to optimize the estimate is to apply the minimum mean square error criterion, i.e.

$$\min E\{S^2(n)\} \tag{2}$$

subject to the constraint

$$E\{S(n)\} = 0 \tag{3}$$

where E denotes the expected value averaged over the population.

To implement the above principles in an algorithm testing environment consider the following. For a given test run, r, let $Z_r^{(k,N)}$ be the average value of Z(n) calculated over a time interval T=NΔt beginning with discrete time instant k, i.e.,

$$Z_r^{(k,N)} = (1/N) \sum_{n=k}^{k+N-1} Z(n) \tag{4}$$

If the number of tests conditions used to evaluate the i-th algorithm is R, then let the average value of $Z_r(k,N)$ for the i-th algorithm be:

$$\text{MEAN} = (1/R) \sum_{r=1}^{R} Z_r^{(k,N)} \tag{5}$$

Ideally the following expression holds:

$$E\{\text{MEAN} - Z_{po}\} = 0 \tag{6}$$

Hence, following can be taken as one measure of algorithm performance.

$$|\text{MEAN} - Z_{po}| \tag{7}$$

The parameter Z(n), for the i-th algorithm, can be expressed as:

$$Z(n) = \text{MEAN} + D(n) \tag{8}$$

where D(n) denotes the deviation of the estimated value from the MEAN. For a decision to be based on an estimated value of Z(n) it is not only important that the MEAN is close to $Z_{po}$, but also that D(n) is small. Hence, an additional measure of algorithm performance for the r-th test run is:

$$\text{STD}_r = [(1/(N-1) \sum_{n=k}^{k+N-1} D^2(n)]^{\frac{1}{2}} \tag{9}$$

If there are R total test conditions applied, then the following provides a second measure of algorithm performance.

$$STD = [(1/(R - 1) \sum_{r=1}^{R} STD_r^2]^{\frac{1}{2}} \qquad (10)$$

Note that these two quantities are proposed as measures of algorithm performance. When using these measures to evaluate the performance of multiple algorithms for a set of test conditions, care must be taken in choosing the value of k, the discrete time instant for beginning the calculation of the measures. The estimates for the i-th algorithm are calculated from $M_i$ consecutive samples, and naturally all must be post fault samples to best estimate $Z_{po}$. This imposes the following constraint on k.

$$k \geq \max_i (M_i) \qquad (11)$$

The proposed criterion function is a linear combination of these performance measures as follows.

$$J = |MEAN - Z_{po}| + a \cdot STD \qquad (12)$$

where the coefficient, 'a', is zero or a positive real number which determines the relative importance of the STD term. Naturally a lower value of J indicates better relative performance.

## Criterion Function Interpretation and Sensitivity

As with all criterion functions of this form, a rationale or set of guidelines is needed for choosing the value of the coefficient 'a'. Consider the following.

a) a = 0: This value may be appropriate when ranking algorithms in which the trip decision is made based on the average of N successive estimates beginning with the k-th sample. In this case, the averaging of successive estimates minimizes the effects of any oscillations in the estimates, and hence, setting a=0 may give the most appropriate criterion function for ranking.

b) a ≠ 0: For relay implementations in which a single or the average of a few successive estimates is used to make the trip decision, a measure of oscillations of the estimates about the mean value is necessary. The criterion function allows greater emphasis to be placed on the stability of the estimates by increasing the value of 'a'.

One rationale for choosing the value of this coefficient is as follows. Assume that the values of D(n) have characteristics of stationary Gaussian noise. Then the characteristics of the Gaussian probability functions can be utilized. For example, the probability is 0.99 that the estimate, Z(n), will lie in the following interval.

$$MEAN - 2.58 \cdot STD \leq Z(n) \leq MEAN + 2.58 \cdot STD \qquad (13)$$

Note in Figure 2 that this means that the estimate should fall in the region between the lines MEAN-2.58·STD and MEAN + 2.58·STD. However, it also means that the maximum distance between the actual value and the estimate is equal to the $|MEAN_i - Z_a| + 2.58 \cdot STD$ with the probability 0.99. Note also that this maximum distance is in fact equal to the value of the criterion with a=2.58. Therefore, the value of the criterion function, with a given probability determined by 'a', is a measure of the distance between the estimated and actual value.

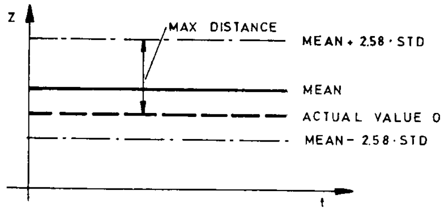

Figure 2. Description of the Algorithm Evaluation Criterion Function

The criterion function is defined by four parameters: the weight coefficient 'a'; the post fault sample, k, in which the first estimate is taken for the time averaging; the number, N, of samples used in the time average; and the number, R, of test conditions or simulation runs in the test set.

The criterion function, defined by specific sets of parameters, was applied to various subsets of the over 10,000 simulation runs made during this study to determine appropriate sets of parameters for different conditions and to determine the criterion function sensitivity to various sets of criterion function parameters, algorithm parameters, and power system conditions. As in the previous case, space does not permit presentation of the complete set of results.

The results of two cases are summarized here: 1) when the criterion function with set parameters was used to rank the algorithms for varying power system conditions; and 2) when the criterion function parameters were varied for given algorithm parameters and power system conditions.

Case 1: Tables VII and VIII show the top five ranked algorithms when the criterion function was applied to Model 1 and Model 2 respectively. The table in Appendix IV gives the criterion function parameters used and the algorithm parameter values. Note that the sampling rate was 16 s/c and the (0.5-1.5)T time interval was under consideration. Hence, k was set equal to 8 and N to 16 so that the criterion function rated the desired conditions.

Note that the four algorithms with a short data window (C1-4, C2-1, c2-2, C2-7) appear in both of the tables. It is interesting to note that the Kalman filtering algorithm (C2-9) performed better for the longer transmission line.

Case 2: Tables IX and X show the top five ranked algorithms when the 'a' parameter of the criterion function was varied. The table in Appendix IV gives a summary of the various parameters which were fixed. Note again that the k and N parameters of the criterion function were set to match the algorithm parameters that were to be tested so that a valid ranking was made.

Table VII. Criterion Function Ranking - Model 1

| Alg. | MEAN | STD | J |
|---|---|---|---|
| C1-3 | 63.93 | 25.19 | 67.70 |
| C1-4 | 63.94 | 25.22 | 67.78 |
| C2-7 | 64.68 | 24.97 | 67.89 |
| C2-2 | 64.55 | 25.36 | 68.75 |
| C2-1 | 64.66 | 25.52 | 69.28 |

Table VIII. Criterion Function Ranking - Model 2

| Alg. | MEAN | STD | J |
|---|---|---|---|
| C2-7 | 60.89 | 23.79 | 61.76 |
| C2-1 | 60.78 | 24.02 | 62.47 |
| C2-9 | 57.02 | 22.83 | 63.16 |
| C2-2 | 64.42 | 24.90 | 67.38 |
| C1-4 | 63.88 | 25.18 | 67.58 |

Table IX. Algorithm Ranking - (0.25-0.75)T Interval

| a = 2.58 | | | | a = 0 | | | |
|---|---|---|---|---|---|---|---|
| Alg. | MEAN | STD | MAX. D. | Alg. | MEAN | STD | J |
| C2-3 | 57.15 | 28.95 | 78.78 | C2-3 | 57.15 | 28.95 | 4.12 |
| C1-3 | 69.90 | 34.25 | 97.05 | C1-4 | 66.73 | 35.99 | 5.46 |
| C1-4 | 66.73 | 35.99 | 98.37 | C2-1 | 67.75 | 36.47 | 6.48 |
| C2-1 | 67.75 | 36.47 | 100.62 | C2-2 | 68.52 | 36.84 | 7.25 |
| C2-2 | 68.52 | 36.84 | 102.34 | C1-3 | 69.90 | 34.25 | 8.63 |

Table X. Algorithm Ranking - (1.5-2.5)T Interval

| a = 2.58 | | | | a = 0 | | | |
|---|---|---|---|---|---|---|---|
| Alg. | MEAN | STD | MAX. D. | Alg. | MEAN | STD | J |
| C2-1 | 61.05 | 0.91 | 2.53 | C2-5 | 61.31 | 4.43 | 0.09 |
| C1-4 | 60.35 | 0.99 | 3.43 | C2-8 | 61.06 | 54.64 | 0.17 |
| C1-3 | 60.35 | 1.09 | 3.69 | C2-1 | 60.98 | 0.91 | 0.18 |
| C2-9 | 63.31 | 0.70 | 3.88 | C2-2 | 60.98 | 1.90 | 0.19 |
| C2-2 | 60.98 | 1.90 | 5.15 | C2-7 | 60.96 | 8.97 | 0.27 |

Note that Table IX contains the same algorithms but in a different order for each value of 'a'. As expected, the maximum distance is much bigger for a=2.58 than for a=0 since the oscillatory behavior of the algorithm estimates receive maximum weight and the test interval is (0.25-0.75)T. One conclusion here is that if the trip decision is to be made using the average of 8 successive estimates, then the a=0 column may be the pertinent ranking.

For the cases in Table X the difference in the maximum distance for a=2.58 and a=0 is not significant. Hence, it is evident that as expected the oscillations are small in the (1.5-2.5)T time interval. An interpretation of these results is that the a=2.58 column indicates that oscillations are small enough that a trip decision might be made on a single estimate or short time average.

## ALGORITHM EVALUATION

To illustrate the potential of the criterion function in determining the 'optimal' algorithm for any given protective relaying application situation, three different application situations were defined and the algorithm evaluation performed.

The first situation was that of a 'long' transmission line with a requirement for a fast relay response time. This translated to a requirement that Model 2 be used, and the time interval be (0.25-0.75)T.

The second situation was that of a 'short' transmission line with a relatively slow response time requirement. This translates to a requirement that Model 1 be used and the time interval considered be (1.5-2.5)T.

The third situation considered was a 'long' transmission line (Model 2) and a medium response time or the (0.5-1.5)T interval.

In the first two cases the assumption was that the trip decision would be made on a single estimate and hence the value of 'a' used was 2.58. In the third case it was assumed that an averaged estimate would be the basis for the trip decision and, hence, 'a' was set to zero. This case also evaluated the effect of no input analog filter.

The results are given in Tables XI, XII and XIII. In each case the most favorable values of sampling rate and data window were first determined for each algorithm using the criterion function. Then these results were ranked according to the criterion function value.

In interpreting Table XI it should be noted that long data window algorithms are at an inherent disadvantage in the (0.25-0.75)T time interval. Note that the Class II algorithms based on the 'optimization' approach (C2-7, C2-9) and the 'fundamental harmonic only' approach (C2-1, C2-3) rank high on the criterion function basis but that the means in the case of C2-3 and C2-9 deviate considerably from the actual value.

Table XI. Algorithm Evaluation - Situation I.

| Alg. | s rate | MEAN | STD | J |
|---|---|---|---|---|
| C2-7 | 32 | 62.36 | 24.67 | 64.84 |
| C2-1 | 24 | 61.67 | 27.26 | 70.73 |
| C2-3 | 32 | 49.93 | 24.41 | 70.81 |
| C2-9 | 24 | 72.43 | 29.84 | 88.16 |
| C1-1 | 32 | 62.97 | 33.87 | 89.10 |
| C1-4 | 16 | 66.95 | 36.01 | 98.60 |
| C2-6 | 12 | 73.07 | 34.81 | 101.62 |
| C2-4 | 16 | 66.40 | 38.27 | 103.88 |
| C2-2 | 16 | 69.03 | 37.54 | 104.62 |
| C1-3 | 12 | 69.00 | 41.70 | 115.33 |
| C1-5 | 32 | 75.17 | 42.04 | 122.36 |
| C2-5 | 96 | 110.36 | 35.40 | 140.41 |
| C1-2 | 12 | 120.25 | 79.32 | 201.31 |
| C2-8 | 12 | 97.52 | 144.68 | 240.90 |

From Table XII it follows that almost all the algorithms exhibit good performance when the interval (1.5-2.5)T is considered. Estimate oscillations are usually minimum in this time interval. Algorithms which ranked high include the minimum square error algorithm with the "backward" derivative approximation and all the Class I algorithms from the integration subclass. Also included are the Kalman filtering algorithm and the C2-1 algorithm.

In Table XIII the STD values indicate that the estimates are very oscillatory in the (0.5-1.5)T interval. However, the MEAN values are close to the actual value as indicated by the relatively small MAX. D. values with a=0. This indicates that there should be acceptable performance if averaged estimates are used in a relay implementation. The first seven algorithms show particularly good performance. This group includes algorithms C2-1, C2-3, C2-4, C2-7, and C2-8 from Class II. Their inclusion in the group for this case was somewhat expected since their estimates

Table XII. Algorithm Evaluation - Situation II.

| Alg. | s rate | MEAN | STD | J |
|---|---|---|---|---|
| C1-1 | 96 | 60.80 | 0.63 | 2.04 |
| C2-9 | 8 | 61.25 | 0.88 | 2.30 |
| C2-1 | 4 | 61.02 | 0.86 | 2.24 |
| C1-2 | 96 | 61.09 | 1.04 | 2.82 |
| C1-4 | 32 | 60.92 | 0.99 | 2.86 |
| C1-3 | 96 | 61.07 | 1.65 | 4.41 |
| C2-2 | 32 | 60.98 | 1.68 | 4.58 |
| C2-7 | 96 | 60.91 | 1.68 | 4.59 |
| C1-5 | 96 | 60.07 | 1.31 | 4.63 |
| C2-5 | 24 | 61.31 | 4.42 | 11.50 |
| C2-4 | 32 | 60.94 | 5.70 | 14.99 |
| C2-6 | 16 | 49.84 | 4.33 | 22.56 |
| C2-3 | 96 | 60.84 | 15.68 | 40.84 |
| C2-8 | 96 | 61.11 | 50.21 | 129.65 |

Table XIII. Algorithm Evaluation - Situation III.

| Alg. | s rate | MEAN | STD | J |
|---|---|---|---|---|
| C2-3 | 96 | 61.39 | 41.74 | 0.12 |
| C2-4 | 16 | 61.05 | 29.17 | 0.22 |
| C2-8 | 96 | 61.68 | 50.31 | 0.41 |
| C2-1 | 16 | 60.79 | 23.62 | 0.47 |
| C1-2 | 96 | 61.75 | 10.42 | 0.48 |
| C2-7 | 16 | 60.78 | 23.74 | 0.48 |
| C1-3 | 96 | 61.84 | 10.23 | 0.57 |
| C1-5 | 96 | 62.52 | 16.08 | 1.25 |
| C2-9 | 32 | 62.67 | 18.39 | 1.40 |
| C1-4 | 32 | 59.69 | 997.66 | 1.58 |
| C2-2 | 24 | 62.97 | 149.46 | 1.70 |
| C2-5 | 96 | 65.48 | 21.09 | 4.21 |
| C1-1 | 24 | 66.43 | 22.10 | 5.16 |
| C2-6 | 32 | 43.83 | 19.23 | 17.44 |

exhibited oscillatory behavior. The two Class I algorithms (C1-2, C1-3) were also expected to be in the group because they exhibited good overall performance at the very high sampling rates.

These evaluations revealed some interesting 'optimal' sampling rates. In Table XI the lowest 'optimal' sampling rate corresponds to that corresponding to the low pass filter cut-off frequency and the sampling theorem. However, Table XII shows two sampling rates (4 s/c and 8 s/c), which are below the Nyquist rate, to be 'optimal' (for C2-1 and C2-9). This indicates that the high frequency content of the signals was not significant. In Table XIII only relatively high 'optimal' sampling rates (≥16 s/c) appear due to the signal components which caused the estimates to be oscillatory.

## CONCLUSIONS

This paper contains the results of a simulation based study. The four main parts are: the definition of the test conditions; the study of algorithm sensitivity to system and algorithm parameters; the definition of an adaptive criterion function for algorithm evaluation; and an evaluation study for three typical digital algorithm applications.

This study has provided a methodology for algorithm evaluation. The methodology is independent of the basis for the definition of any particular algorithm and, hence, is applicable to the evaluation of all algorithms which have been proposed. The test cases employed in the study were all based on models of 138 kV lines. Additional analysis and testing with additional models and voltage levels will be necessary to better explain some of the properties identified here. The authors intend to continue their study to obtain these answers.

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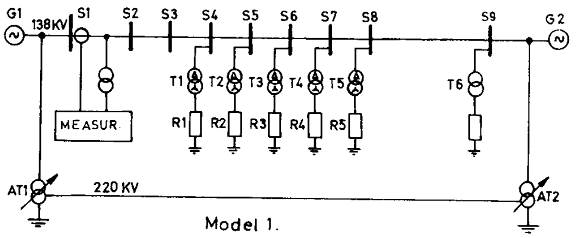
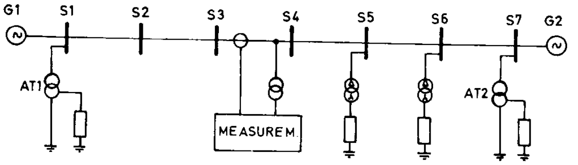
## APPENDICES

### APPENDIX I: Modeling Parameters



Model 1.



Model 2.

| GENERATORS | $U_f$ [V] | $Z_f$ [Ω] | f [Hz] | $\varphi(f)$ [DEG] $\varphi_1(t)-\varphi_2(t)=1°$ |
|---|---|---|---|---|
| $G_1$ | 114708 | .26+ j2.6 | 60 | $\varphi_1(t)$ |
| $G_2$ | 119452 | .6+j6 | 60 | $\varphi_2(t)$ |

| TRANSM. LINE MODELS | $R_S$ [Ω/mile] | $R_M$ [Ω/mile] | $X_S$ [Ω/mile] | $X_M$ [Ω/mile] | $C_S$ [nF/mile] | $C_M$ [nF/mile] | LENGTH (mile) |
|---|---|---|---|---|---|---|---|
| MODEL 1 | 0.276 | 0.135 | 1.326 | 0.652 | 13.94 | -2.59 | 27.93 |
| MODEL 2 | 0.245 | 0.150 | 1.320 | 0.658 | 14.12 | -2.98 | 122.00 |

### APPENDIX II: Test Conditions

| MODEL 1. | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TYPE OF FAULT | AG | | | | | | | | ABG | | | | | | | | ABC | |
| FAULT DISTANCE [%] | 11 | | 68,7 | | 90,5 | | 100 | | 11 | | 68,7 | | 90,5 | | 100 | | 11 | |
| FAULT ANGLE [DEG] | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 |
| $f_N=60$ Hz | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| TOTAL No. OF FAULTS | 18 | | | | | | | | | | | | | | | | | |

| MODEL 2. | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TYPE OF FAULT | AG | | | | | | | | | | ABG | | | | | | | | ABC | | | |
| DISTANCE [%] | -11 | | 11 | | 68,7 | | 90,5 | | 100 | | 11 | | 68,7 | | 90,5 | | 100 | | -11 | | 11 | |
| ANGLE [DEG] | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 | 0 | 90 |
| $f_N$ (±1%; ±5%) | | | | | | | | | | | | | 4 | | | | | | | | | |
| RESISTANCE | | | 1 | | | | | | | | | | 1 | | | | | | | | | |
| PAR. COMP. | | | | | | | | | | | | | 1 | | | | | | | | 1 | |
| SER. COMP. | | | | | | | | | | | | | 1 | | | | | | | | 1 | |
| $f_N=60$ Hz | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| TOTAL | 32 | | | | | | | | | | | | | | | | | | | | | |

### APPENDIX III: Sensitivity Study Test Conditions

| SENSIT PARAM / TEST COND. | FREQUENCY | LINE COMP | LOW PASS. F. | SAMP. RATE | DATA WINDOW |
|---|---|---|---|---|---|
| SYSTEM MODEL | 2 | | | 1 | |
| SYS. FREQ. | 57 - 63 | 60 | | | |
| FAULT RESIST. | 0 | | | | |
| LINE COMP. C | NO | 69 nF | NO | | |
| FAULT LOCAT | 68.7 | 11<br>68.7 | 11 ÷ 100 | | |
| FAULT TYPE | ABG | ABC, ABG | ALL | | |
| FAULT ANGLE | 0 | | ALL | | |
| SAMP.RATE[C/S] | 16 | 24; 32 | 16; 32 | 4 ÷ 96 | 16; 32 |
| FILTER (ORDER) | 3 | | 0 AND 3 | 3 | 1 |
| DATA WINDOW | VARIED | | | | |
| TIME INT. | 3 | 1 AND 3 | 2 | 3 | 2 |

### APPENDIX IV: Evaluation Study Test Conditions

| TABLE NO | CRITERION ENTITIES | | | | MODEL | S. RATE [S/C] | TIME INT. | FILTER (ORDER) |
|---|---|---|---|---|---|---|---|---|
| | ε | N | K | R | | | | |
| VII | 2.58 | 16 | 8 | 18 | 1 | 16 | 2 | 3 |
| VIII | 2.58 | 16 | 8 | 18 | 2 | 16 | 2 | 3 |
| IX | 2.58; 0 | 8 | 4 | 18 | 1 | 16 | 1 | 3 |
| X | 2.58, 0 | 16 | 24 | 18 | 1 | 16 | 3 | 3 |
| XI | 2.58 | S/2 | S/4 | 18 | 2 | S-OPT. | 1 | 3 |
| XII | 2.58 | S | 3S/2 | 18 | 1 | S-OPT. | 3 | 3 |
| XIII | 0 | S | S/2 | 18 | 2 | S-OPT | 2 | NO |

OTHERCONDITIONS: FREQUENCY - 60Hz; FAULT RESISTANCE - 0, NO CAP. COMPENSATION; FAULT LOCATION (%) - 11; 68.7, 90.5, 100; TYPE OF FAULT - AG, ABG, ABC,; FAULT ANGLE (°) - 0,90;

## Discussion

**G. D. Rockefeller** (Rockefeller Associates, Inc., St. Rose, LA): The authors considered sampling rates up to 96/cycle. Rates above about 24/cycle appear impractical because of A/D accuracy limitations unless dynamic ranging is used. Relaying requires a dynamic range of about 8000/1 (a 13-bit word, plus sign bit). With signals at the low end of the range, differencing errors involving closely spaced samples become quite significant.

The authors did not compare the computational burdens. This seems proper, since advances in microprocessor technology have relegated to a secondary level concerns that were much more significant in the 1970's.

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**J. S. Thorp** (Cornell University, Ithaca, NY) and **A. G. Phadke** (Virginia Tech, Blacksburg, VA): The authors have attempted a task which in our view is difficult, and in fact some of the results they report seem completely at odds with our own experience. We have often wondered whether comparison of distance relaying algorithms can ever be done in an objective and comprehensive manner. In our view, the results of this paper support the conclusion that it is futile to compare algorithms, because there is danger of reaching totally wrong conclusions from a superficial analysis of the algorithms.

One difficulty is that before the algorithms could be evaluated, the authors would have to understand all the algorithms in great detail. We are very familiar with the algorithm C2-8 of the paper, and think that at least in this case, the evaluators have missed many details and have thus obtained totally wrong results with this algorithm. This algorithm has been working satisfactorily in a field environment since 1980. It could not possibly have produced the kinds of errors in distance estimates as reported by the authors, and still survive as a relay in the field. We would like to offer some details of this algorithm which the authors may not have reckoned with in their study. See [A] and [B].

The algorithm needs a mimic circuit for it to function properly. At present the mimic circuit is analog, although a digital mimic realization is possible. In any case, a proper representation of the mimic circuit is essential in any evaluation of this algorithm. Further, the algorithm uses a transient-monitor function as a window to block a trip decision when conditions warrant it. The algorithm also functions with a variable data window. None of the features are evident in the description of algorithm C2-8 in the paper. Not using the mimic in a test of this algorithm would render the results of such a study meaningless.

We are also struck by the authors' treatment of the anti-aliasing filters. The paper implies that although sampling rates of 4, 8, 12, 16, 24, 32, and 96 samples per cycle were used, only three types of anti-aliasing filters were used: the third-order Butterworth with cut-off frequency of 320 Hz; a first-order filter with cut-off at 800 Hz; and (see Tables V and VI of the paper) no anti-aliasing filter at all. At least as far as our work is concerned, a proper (i.e., with a cut-off frequency equal to one-half the sampling frequency) anti-aliasing filter should be included for each sampling frequency in order to make valid comparisons. Why have the authors made comparisons with intentionally aliased samples?

Regarding the system used for comparison studies, we feel that the authors have left out one of the most significant effects which causes dispersion of distance estimates. It is shown in (17) that the complexity of the source system has an impact on the nature of nonfundamental frequency components of fault current and voltage waveforms. The model shown in Appendix I has only source inductances at each end—there are no shunt capacitances which would be contributed by transmission lines feeding the buses where the relays are located. Since the actual service condition of a relay must cope with such (more complex) system configurations, comparisons made with a very simple source structure are not conclusive.

We also wonder what is the significance of calculating the mean and standard deviation over a range of samples ($k - N$). If a relay makes its decision at sample $k$, it is the scatter of estimates at $k$ which is of interest from the relaying point of view. Any computations performed at later samples are simply an input to some fault-location function. In our opinion, the mean, standard deviation, and the index $J$ are inappropriate measures of an algorithm's performance as a relay.

Finally, we wonder whether in performing these evaluations the authors considered getting some clarifications from developers of algorithms. If the standard deviation of an impedance estimate turns out to be greater than its mean with a particular algorithm, it should be a cause for concern that perhaps the algorithm is not being tested properly.

We believe this paper serves the purpose of alerting future workers that algorithm comparison is a hazardous task at best.

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**Allan T. Johns** (The City University, London, United Kingdom): This is in my opinion an interesting paper but nevertheless one which I find rather misleading because, put quite simply, an "algorithm" is not a digital relay. In fact, the algorithm itself is but one small part of a digital relaying system and its influence in terms of overall performance is often correspondingly small; this perhaps accounts for there being diminishing activity and interest in work aimed at comparing the response of various algorithms.

My own experience, which is based largely upon that gained in engineering actual digital protection relays and evaluating their performance from site trial data performed on the British 400-kV system, is that the performance of relays in practice tends to be influenced most by the following factors:

a) The system and relay transducer/analog interfaces.

b) The presence of noise (particularly that within the bandwidth of the relay and superimposed upon the measuring signals). This is particularly acute in some substation environments and often demands the use of signal level threshold techniques incorporated within and/or outside the algorithm itself.

c) The means by which the relay retains directional discrimination, particularly where capacitor voltage transducers are involved.

d) The analog/digital conversion word lengths used at various points throughout the digital processing.

As the paper makes little or no reference to the above considerations, I find it difficult to interpret the results in a meaningful way. Indeed, my own experience leads me increasingly towards the view that it is fruitless to attempt to attach any real practical significance to algorithm-ranking type exercises of the type suggested in the paper. For example, an algorithm may have very good noise rejection qualities but might otherwise fare poorly according to the criteria adopted. Similarly, yet another set of criteria produce different conclusions and ranking. While accepting that it might not have been the authors' primary intention to rank the algorithms considered, one could justifiably be forgiven for assuming that this was at least one primary use of such an exercise. Notwithstanding the foregoing considerations, an actual relay must operate satisfactorily over a very wide range of system frequency conditions corresponding to system impedance ratios ranging from perhaps 0–60; for any such exercises to be meaningful due care needs to be taken of this factor also.

I would find it useful to have the authors' considered views on the significance of the results they have presented particularly when set against the above summarized practical considerations.

Manuscript received February 24, 1987.

**M. Kezunovic, S. Kreso, J. T. Cain,** and **B. Perunicic**: The authors would like to thank the discussers for their interest in the paper.

Mr. Rockefeller's comment about practicality of the sampling rates above 24 s/c, because of the A/D conversion accuracy, is very interesting. A similar question related to A/D conversion word length was also raised by Dr. Johns. In our opinion this problem is technology dependent. It is felt that this will not be a practical problem in the future when hardware development will provide for inexpensive, high resolution, high speed A/D converters. Coupled with the dynamic ranging, the A/D accuracy limitations should not be a problem. That is why this issue was not considered in our study.

General comments by Drs. Thorp, Phadke, and Johns have implied that the study given in the paper is of a questionable practical value. The main argument of Drs. Thorp and Phadke is the "difficulty" of this task which in their opinion "prevents an objective and comprehensive" algorithm study. Dr. Johns considers the paper "rather misleading" since the "algorithm" is only a "small portion of a digital relay design and its influence in terms of overall performance is often correspondingly small." The study reported in the paper has been performed as a part of an overall research and development activity that resulted in implementation of a distance relay [1], [2]. Our experience is different from the one of the discussers. We also found the algorithm study to be a difficult task, but we did not find that this has prevented us from reaching some objective and comprehensive results. We hope that the study results given in the paper illustrate this. As per the comment of Dr. Johns, we agree that the "algorithm" is only a small portion of a digital relay design, but we do not agree that its influence is correspondingly small. If the influence was small, then this may imply that any algorithm can do the job as well as any other algorithm. We feel that this would be definitely a wrong conclusion. As an outcome of the general comments, another question is posed in our mind: what is the alternative approach to the algorithm study that would have a practical value? Unfortunately, the discussers did not comment on this. From our experience, algorithm comparison and evaluation cannot be avoided in any systematic and comprehensive digital relay design activity. However, this does not mean that the final relay design evaluation should be based on this study alone. It is only appropriate to form the final judgment after a thorough testing of the final relay design is completed. But, initial understanding of the algorithm comparative performance is quite important when implementation decisions are to be made.

Drs. Thorp and Phadke carry on with their argument by indicating that we "have missed many details of algorithm C2-8, and thus obtained totally wrong results with this algorithm." The following discussion indicates that what Drs. Thorp and Phadke call algorithm C2-8, as given in their cited references, is not exactly the same as the algorithm that we have designated as algorithm C2-8 given in Table I of the paper. Regardless of the issue of the algorithm C2-8, it is felt that the conclusion, that only one algorithm out of 14 algorithms considered is not treated appropriately, should not be a sufficient argument to designate the whole study as yielding "totally wrong conclusions." On the other hand, we would like to provide some additional comments that indicate that algorithm C2-8 was treated fairly and correctly and hence their comments seem even more surprising.

First, we have performed a study of the *basic* algorithms used for calculation of the critical relaying quantities, namely line parameters $R$ and $X$, or phasors $V$ and $I$. This is clearly indicated in Table I given in the paper. It is obvious that we have not studied the algorithms that could be developed to accommodate the entire function of protective relaying including various adjustments made in the basic algorithm for the purpose of performance improvements. There are two major reasons for such an approach. The first reason is methodological in nature. It is reasonable to study the basic algorithms first, in order to make further conclusions about a possible need for any adjustments related to the performance improvements. The other reason is related to the difficulty in defining a fair testing set-up, if each algorithm has its own set of adjustments for performance improvements. Taking this explanation into account, it is obvious that our testing conditions and the basic definition for algorithm C2-8 were quite different from the testing conditions and the definitions considered by Drs. Thorp and Phadke in the cited references. Our tests have identified the situations when the basic algorithm needs some adjustments for performance improvements (situations I and II), and when the adjustments are not needed (situation III). Incidentally, those conclusions are in full agreement with the theoretical considerations of the expected performance. Therefore, we do not see how our results can be considered to be "totally wrong" when they actually confirm the need for the approach that Drs. Thorp and Phadke have undertaken.

An illustration of the situation described above is our treatment of the mimic circuit problem. Our choice was not to take into account the mimic circuit simulation because we consider this to be an adjustment which is not assumed in the basic algorithms definition, as given in Table I. The algorithm C2-8 ranked very high in Situation III (Table XIII) and very low in Situations I and II (Tables XI and XII). This illustrates that algorithm C2-8 needs some adjustments to perform "better" in situations I and II. Drs. Thorp and Phadke had shown in their references what are the needed adjustments and what is the performance of the overall algorithm using these adjustments. Therefore, the results of our study are not "meaningless" because they indicate the need for adjustments. Since one of the adjustments was the use of the mimic circuit, it may be very logical to perform our tests taking into account this feature. However, this would also mean that we should test all of the 14 algorithms with this adjustment added. This would have been an extension of our study since in that case adjustments for the basic algorithms would have to be tested. Our goal was to test the basic algorithms only, since the results of this study give a very good guidance about further algorithm adjustments. On the other hand, if all possible algorithm adjustments are tested on all 14 algorithms, the results might be meaningless since some algorithms do not need any adjustments for the given situations.

As per our treatment of the anti-aliasing filter, we would like to further clarify our approach. During the course of the theoretical study of the basic algorithms it was noticed that one of the main estimation problems was to reconstruct the line parameters using signal samples rather than to reconstruct the original signals. Hence, the requirement of a necessary and sufficient condition of the Nyquist sampling rate for an algorithm for the given antialiasing filter cut-off frequency could be questioned. This is why the given sensitivity study regarding the low-pass filter cut-off frequency was performed. The results have confirmed that this requirement might not be as critical as considered by a number of researchers in the past. On the other hand, we are very much aware of the sensitivity of various algorithms to the sampling rate selection. Our final evaluation results (Tables IX, XII, and XIII) are obtained selecting the optimal sampling rate for each algorithm, i.e., the sampling rate for which the algorithm had shown its best performance.

Drs. Thorp and Phadke have suggested that, in order to make a more conclusive study, a more complex system simulation model should be used. We completely agree, and our further study is along those lines. However, we still believe that the system simulation model that was used is sufficient to identify some major properties of the basic algorithms.

We also believe that our use of the time averages as a substitute for the population averages, in the case when algorithm decision is based on an estimate at sample K, is acceptable since this should not significantly affect the statistical properties of the mentioned estimates.

Finally, the references that we have used were sufficient for us to understand the basic algorithms. However, for an evaluation study that would be related to the overall relaying algorithm comparison, it would be necessary to contact the authors for clarifications needed regarding various adjustments made for performance improvements. Since our intention is to pursue this course of study, we would appreciate future cooperation of all of the authors.

In closing our discussion, regarding comments of Drs. Thorp and Phadke, we hope that the additional explanations have indicated that "their" algorithm C2-8 has been treated fairly and correctly in our study and that our conclusions are not "at odds" with theirs. We admit that our use of the authors' names in Table I in the paper, along with the basic algorithms that they invented can be misleading. This approach may confuse the issue of validity of our conclusion regarding the performance of the basic algorithms vs. the conclusion reached by the authors regarding the performance of the modified algorithms used in actual relay implementation. In this sense we appreciate very much the comments of Drs. Thorp and Phadke since we were given an additional opportunity to clarify our approach.

Regarding practical considerations given by Dr. Johns, we can only say that we agree that his points are important. But this does not make us believe that our considerations given in the paper are not practical and important as well. Even though our study was not aimed at resolving the specific problems mentioned by Dr. Johns, the methodology provided in our study can be used to analyze some of these problems. The relay transducer interface can be considered as an analog filter. It can be modeled and added to the simulation package that we have used [3]. A similar approach can be taken to study the capacitor voltage transducer in particular [4]. As per consideration of the signal noise influence, our study indicates the performance of the basic algorithms for different types of input signals (Model I and Model II). An additional analysis may reveal what was the performance of the algorithms that take the noise consideration into algorithm definition vs. the ones that are not based on this assumption. Incidentally, our present work is along the lines of the practical considerations indicated by Dr. Johns.

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