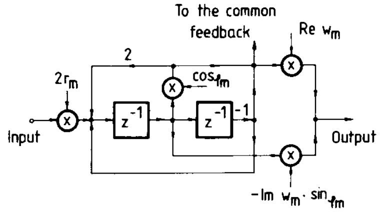

Fig. 2.

In the following we will show that the positioning strategy proposed by condition (10) (see also (11)) has very interesting consequences. First in Fig. 2 we present a simple second-order structure, which "structurally" forces the fulfilment of condition (10), and at the same time is canonic considering the number of delay elements and the number of nontrivial multiplications. Using this structure the resonator poles will be always on the unit circle, coefficient quantization will affect only the "angle" of these poles. It is important to note that this second-order section implements

$$G_m(z) = w_m H_m(z) + w_m^* H_m^*(z) \tag{12}$$

where asterisk denotes a complex conjugate.

As a next step let us investigate the behavior of the common feedback loop. The transfer function from the input to point P (see Fig. 1) has the following form:

$$H_P(z) = \frac{\sum_{n=0}^{N-1} H_n(z)}{1 + \sum_{n=0}^{N-1} H_n(z)}. \tag{13}$$

It is very easy to show that the magnitude of this transfer function is less or equal to the unity (i.e., it is "passive" in this sense), if

$$\mathrm{Re} \sum_{n=0}^{N-1} H_n(z) \geqslant -\frac{1}{2}. \tag{14}$$

Condition (14) can be fulfilled at every frequency if condition (10) holds, and

$$\sum_{m=0}^{N-1} \mathrm{Re}\left[\frac{g_m}{z_m}\right] = \sum_{m=0}^{N-1} r_m \leqslant 1. \tag{15}$$

The proof is straightforward, and will be omitted here. Equality condition in (15) is easily achieved if we have one more resonator pole than filter pole. This is due to the fact that the coefficient of z^{-N} in the denominator polynomial of $H(z)$ will be zero if in (15) equality holds. In this case $H_P(z)$ implements an all-pass filter, and if so, we know even its zeros, since they are in mirror image relationship with the poles of $H(z)$.

At this point of our development we can determine those resonator pole positions which will provide the above properties. These positions coincide with the zeros of $1 - H_P(z)$, since the input of the resonators (see point C in Fig. 1) is the difference of the filter input and the output at point P. We will have two sets of resonator poles, since the filter poles do not specify the sign of $H_P(z)$.

If the number of resonator poles equals the number of the filter poles, $H_P(z)$ cannot be an all-pass transfer function, because it is forced to have at least one zero at the origin, otherwise the loop would be delay free. The resonator pole positions, however, can be determined rather similarly by finding the zeros of $1 - H_A(z)$, where $H_A(z)$ is an all-pass function having the same poles as the filter has. These resonator poles will meet condition (10), and one of the two resonator pole sets will insure also the fulfillment of condition (15).

In the case of the recursive Fourier transformation $r_m = 1/N$ ($m = 0, 1, \cdots, N-1$), thus condition (15) is fulfilled, and $H_P(z) = z^{-N}$, that not surprisingly will provide as resonator poles the Nth roots of the unity. The companion set consists of the Nth roots of -1, and generates a "companion" recursive Fourier transformation that can play some role, if the zero frequency component of the signal should be suppressed.

IV. Conclusions

In this paper the sensitivity properties of a recently introduced resonator-based structure has been presented. This structure, from some respects of the filter design, is closely related to the frequency-sampling structure, however, due to a common feedback, has low sensitivity to the coefficients. The common feedback provided perfect pole–zero cancellation, thus the application of ideal resonators does not cause implementational problems. It is also shown that the resonator poles can be arbitrarily located on the unit circle, however, there exists a strategy which results in a canonic solution, and insures the "passivity" of the common feedback loop.

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Bilinear Form Approach to Synthesis of a Class of Electric Circuit Digital Signal Processing Algorithms

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Abstract **—A number of different digital signal processing algorithms for Electric Power System data acquisition, control and protection were introduced in the past. Each of the algorithms was defined based on the specific application utilizing either some heuristic approaches or known systems identification and parameter estimation techniques. A generalized methodology for algorithm synthesis which may be used in a number of different applications is proposed in this paper based on the Bilinear Form approach.**

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I. Introduction

The main topic of this paper is a class of digital signal processing algorithms used for calculation of electric circuit parameters and signal quantities. Of particular interest are algorithms used to calculate Electric Power System active and reactive power, frequency and transmission line parameters. It should be recognized that this class of algorithms has a wide range of application in various control and monitoring functions [1].

The previous research activities in the field have been concentrating on definition of the specific digital signal processing algorithms to be applied to a particular problem. Besides being application oriented, the early algorithms were defined mostly using some heuristic approaches. As the application prospects broadened, some classical parameter estimation and system identification techniques were applied to this field. These techniques were used to estimate values of $\boldsymbol{v}$ and $\boldsymbol{i}$. Then, some classical electric circuit relations were used to calculate circuit parameters or quantities such as active power, reactive power, rms values, system frequency. Therefore, a different signal processing technique may be selected for the same application constraints. However, there is no clear methodology and/or criterion available as a guideline for selecting the appropriate technique. Furthermore, there is no common mathematical tool available for different algorithm comparison and evaluation. This paper provides a definition of a generalized mathematical representation of the mentioned algorithms using Bilinear Form of signal samples. The basic steps of the algorithm synthesis methodology are also outlined.

II. Bilinear Form of Harmonic Signal Samples

The bilinear form of two sequences of samples x_n and y_n calculated in the discrete time $\boldsymbol{n}$ and denoted as BF_n is given by the following expression [2]:

$$\mathrm{BF}_n = \sum_{k=0}^{N} \sum_{m=0}^{N} h_{km} x_{n-k} y_{n-m}. \tag{1}$$

The coefficient h_{km} is a weight attached to the product of two delayed samples x_{n-k} and y_{n-m}. The quadratic matrix $\boldsymbol{H}$ defining the bilinear form:

$$\boldsymbol{H} \triangleq \{ h_{km} \}$$

will be denoted the *weight matrix*. Its dimension is $(N+1)\times(N+1)$ for a window having the width equal $N \cdot \Delta t$.

Let us assume that the samples of two harmonic signals are defined as:

$$x_n = X\cos(n\psi + \phi)$$

$$y_n = Y\cos n\psi \tag{2}$$

where

X, Y	signal magnitudes,
ϕ	phase between two signals,
$\psi = 2\pi\omega_0/\omega_s$	electrical angle between two samples,
ω_0	system fundamental frequency,
ω_s	sampling frequency.

In this case it can be shown that the value of the bilinear form may be expressed as a sum of a constant term BF^c and a variable term BF_n^v:

$$\mathrm{BF}_n = \mathrm{BF}^c + \mathrm{BF}_n^v. \tag{3}$$

These two terms depend on two functions defined on the weight matrix. The first function determines the constant part and is denoted as $H^c(p)$:

$$H^c(p) = \sum_{r=-N}^{+N} h_r^c \cdot p^r \tag{4}$$

where

$$h_r^c = \sum_k \sum_m h_{km}, \qquad 0 \leqslant k \leqslant N$$
$$k - m = r, \qquad 0 \leqslant m \leqslant N. \tag{5}$$

The other function determines the time dependent variable part and is denoted as $H^v(p)$:

$$H^v(p) = \sum_{r=0}^{2N} h_r^v \cdot p^r \tag{6}$$

where

$$h_r^v = \sum_k \sum_m h_{km}, \qquad 0 \leqslant k \leqslant N$$
$$k + m = r, \qquad 0 \leqslant m \leqslant N. \tag{7}$$

Using these functions the constant and the variable term can be expressed, respectively,

$$\mathrm{BF}^c = \frac{XY}{2} \left| H^c(e^{-j\psi}) \right| \cdot \cos\left[\phi + \angle H^c(e^{-j\psi})\right] \tag{8}$$

$$\mathrm{BF}_n^v = \frac{XY}{2} \left| H^v(e^{-j\psi}) \right| \cdot \cos\left[2nd + \phi + \angle H^v(e^{-j\psi})\right]. \tag{9}$$

In this manner the design of the weight matrix $\boldsymbol{H}$ for a particular value of ψ reduces to the choice of suitable functions $H^v(p)$ and $H^c(p)$.

III. Specific Values of Bilinear Forms

If (9) is analyzed it can be seen that the variable term has an average equal to zero and a period equal to the one half of the fundamental frequency harmonic. This term will vanish if the following condition is fulfilled:

$$H^v(e^{-j\psi}) = 0. \tag{10}$$

This is the case when $e^{-j\psi}$ is a zero of the polinominal $H^v(p)$. The variable term will vanish for any ψ if $H^v(p)$ is indenticaly equal to zero. This is the case when:

$$h_r^v = 0, \qquad r = 0, 1, \cdots, 2N. \tag{11}$$

Geometrically, the condition (11) means that the sums of matrix elements in anti-diagonal and all the sub-anti-diagonals have to be zero. Such matrices will be named *constant-valued* [2]. The Bilinear Form defined by a constant-valued weight matrix will have a constant value regardless of the system and sampling frequency. The constant term BF^c depends on product of two phasors' magnitudes and their mutual phase shift. If the signal $x(t)$ is the voltage and $y(t)$ is the current of the same circuit, then the constant term may be used to calculate active and reactive power.

IV. Digital Signal Processing Algorithms for Power and Line Parameter Calculations

The active power is obtained if

$$H^c(e^{-j\psi}) = 1. \tag{12}$$

The reactive power is obtained if

$$H^c(e^{-j\psi}) = -j. \tag{13}$$

If the two signals are equal $x_n = y_n$, the form will be quadratic, and the constant term value will be

$$QF^c = \frac{X^2}{2}\operatorname{Re} H^c(e^{-j\psi}). \tag{14}$$

The square of rms value will be obtained if

$$\operatorname{Re} H^c(e^{-j\psi}) = 1. \tag{15}$$

Obviously, the weight matrices suitable for active power calculation may be also used for rms calculation. Furthermore, let us show how the weight matrices suitable for active and reactive power calculation may be used for the line parameter calculation.

The resistance and reactance of a line may be expressed in terms of P, Q and $I^2/2$ as follows:

$$R = \frac{V}{I}\cos\phi = \frac{\frac{VI}{2}\cos\phi}{\frac{I^2}{2}} = \frac{P}{(RMSI)^2} \tag{16}$$

$$\omega_0 L = \frac{V}{I}\sin\phi = \frac{\frac{VI}{2}\sin\phi}{\frac{I^2}{2}} = \frac{Q}{(RMSI)^2} \tag{17}$$

In this manner the calculation of line parameters reduces to the calculation of active power, reactive power and square of magnitude [3].

V. Selection of the Suitable Weight Matrices

It will be shown how properly scaled skew-symmetric and symmetric matrices may be used to construct the suitable weight matrices [2].

Let us consider first skew-symmetric matrices defined by

$$\boldsymbol{B}^T = -\boldsymbol{B} \tag{18}$$

It can be shown that for such matrices the following holds:

$$\operatorname{Re} B^c(e^{-j\psi}) = 0 \qquad \forall\psi \tag{19}$$

and

$$B^v(e^{-j\psi}) = 0 \qquad \forall\psi \tag{20}$$

Now, if the following condition is also satisfied:

$$\operatorname{Im} B^c(e^{-j\psi}) \neq 0, \qquad \psi = \psi_0 \tag{21}$$

then a weight matrix for reactive power calculation can be constructed as

$$\boldsymbol{H}_Q = \frac{1}{\operatorname{Im} B^c(e^{-j\psi_0})}\cdot \boldsymbol{B} \tag{22}$$

Let us consider now symmetric matrices defined as

$$\boldsymbol{A} = \boldsymbol{A}^T. \tag{23}$$

It can be shown that

$$\operatorname{Im} A^c(e^{-j\psi}) = 0, \qquad \forall\psi \tag{24}$$

These matrices appear as candidates for active power calculation. However, they are not necessarily constant-valued as the skew-symmetric matrices. One way to construct a constant-valued symmetric matrix $\boldsymbol{A}$ is to choose its elements to satisfy the following conditions:

$$\sum_k\sum_m a_{km} = -\frac{a_{rr}}{2}, \qquad r = 0,1,2,\cdots N$$

$$0 \leqslant k \leqslant N, 0 \leqslant m \leqslant N, \qquad k > m,\ k + m = 2r$$

$$\sum_k\sum_m a_{km} = 0, \qquad r = 0,1,2,\cdots N-1$$

$$0 \leqslant k \leqslant N, 0 \leqslant m \leqslant N, \qquad k > m,\ k + m = 2r+1 \tag{25}$$

If the following condition is also satisfied:

$$\operatorname{Re} A^c(e^{-j\psi_0}) \neq 0 \tag{26}$$

then a weight matrix for real power calculation can be constricted as

$$\boldsymbol{H}_P = \frac{1}{\operatorname{Re} A^c(e^{-j\psi_0})}\cdot \boldsymbol{A}. \tag{27}$$

An example of a pair of so obtained weight matrices is given below:

$$\boldsymbol{H}_P = \frac{1}{4\sin^2\psi}\begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{H}_Q = \frac{1}{2\sin^2\psi}\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}. \tag{28}$$

VI. Weight Matrices Providing a Low Sensitivity to System Frequency Changes

Here we will describe a case where the given approach can be used for synthesis of a digital algorithm for power calculation under certain sensitivity constraints.

The system frequency ψ_0 is constant under the normal operating conditions. However, there are some situations where this frequency will vary. As a rule, these changes are not big. Using the expression given in (3), one could derive the conditions for making the influence of the frequency change as small as possible. The value of the bilinear form will be only slightly changed for small deviations of $\psi - \psi_0$ if the following conditions are satisfied:

$$\frac{d}{d\psi}H^c(e^{-j\psi}) = 0, \qquad \psi = \psi_0 \tag{29}$$

$$\frac{d}{d\psi}H^v(e^{-j\psi}) = 0, \qquad \psi = \psi_0. \tag{30}$$

If matrix $\boldsymbol{H}$ is constant-valued then the condition (30) is always satisfied. Also if $\boldsymbol{H}$ is symmetric or skew-symmetric then the condition (29) reduces to

$$\frac{d}{d\psi}\operatorname{Re} H^c(e^{-j\psi}) = 0, \qquad \psi = \psi_0 \tag{31}$$

for active power calculation and to the following condition:

$$\frac{d}{d\psi}\operatorname{Im} H^c(e^{-j\psi}) = 0, \qquad \psi = \psi_0 \tag{32}$$

for reactive power calculation.

An example of the matrices for power calculation with low sensitivity to fundamental frequency changes are given below:

$$H_P = \begin{bmatrix} 0 & 0 & a & b \\ 0 & -2a & -b & 0 \\ a & -b & 0 & 0 \\ b & 0 & 0 & 0 \end{bmatrix}, \quad a = \frac{\cos 2\psi_0 + \cos^2 \psi_0}{4\sin^4 \psi_0}, \quad b = -\frac{\cos \psi_0}{4\sin^4 \psi_0}$$

$$H_Q = \begin{bmatrix} 0 & -a & -b \\ a & 0 & 0 \\ b & 0 & 0 \end{bmatrix}, \quad a = -\frac{\cos 2\psi_0}{2\sin^3 \psi_0}, \quad b = \frac{\cos \psi_0}{4\sin^3 \psi_0} \tag{33}$$

VII. Conclusions

It has been shown that a bilinear form of signal samples may be used to define different algorithms for electric circuit signal and parameter calculation. A generalized form of the algorithm is defined using the weight matrix representation. Some specific algorithms are derived by selecting an appropriate weight matrix. This approach provides mathematically consistent methodology for algorithm synthesis.

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