Online Optimal Transmission Line Parameter Estimation for Relaying Applications

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***Abstract*—Transmission line protective relaying algorithms usually require transmission line parameters as inputs and thus accuracy of line parameters plays a pivotal role in ensuring the reliable performance of relaying algorithms. Online estimation of line parameters is highly desirable and various methods have been proposed in the past. These methods perform well when the measurements utilized are accurate; they may yield erroneous results when the measurements contain considerable errors. Based on nonlinear optimal estimation theory, this paper puts forward an optimal estimator for deriving the positive sequence line parameters, capable of detecting and identifying the bad measurement data, minimizing the impacts of the measurement errors and thus significantly improving the estimation accuracy. The solution is based on the distributed parameter line model and thus fully considers the effects of shunt capacitances of the line. Case studies based on simulated data are presented for demonstrating the effectiveness of the new approach.**

***Index Terms*—Bad measurement detection and identification, distributed parameter line model, non-linear estimation theory, transmission line parameter estimation.**

I. Introduction

TRANSMISSION line parameters including series resistance, series reactance, and shunt susceptance are critical inputs to diverse power system analysis programs. Precision of line parameters is thus essential in ensuring the accuracy of the obtained analysis results. Especially in areas of power system protection, many distance relaying algorithms entail line parameters for determining appropriate relay settings, calculating fault distance, and making a sound tripping decision [1]–[3]. It has been established that the value of line resistance, inductance, and capacitance is frequency dependent [4]–[6]. This paper is concerned only with estimating line parameters at the fundamental frequency, which are usually required by relaying applications.

Various algorithms for computing transmission line parameters have been presented in the past literature. Classical approaches as described in [7]–[9] utilize factors such as tower and conductor geometric parameters, conductor type, assumed ambient conditions, etc. for estimating line parameters. Since actual values of these factors may differ from those employed in the estimation process, considerable discrepancies between actual line parameters and estimated values could occur.

An alternative approach for line parameter estimation is discussed in [10], where the line parameters are derived based on impedances calculated at one terminal using acquired voltage and current phasors assuming the other end of the line is open or short-circuited. Such required measurements may be difficult to obtain. Moreover, inaccuracies in the measurements may lead to appreciable errors in parameter estimation.

To obtain the most prevailing line parameters, online estimation approaches would be highly desirable and especially beneficial to protection applications. This type of techniques take advantage of online voltage and current measurements [11]–[15]. In [11], two sets of synchronized voltage and current phasors from the two terminals of the line are utilized to obtain the ABCD parameters of the line. How distributed parameter per unit length can be obtained is not covered. The estimation accuracy highly relies on synchronization precision [12]. The authors of [13] suggest a method for deriving the line characteristic impedance and propagation constant by making use of online voltage and current phasors captured at sending-end and receiving-end of the line. Reference [14] solves for the line parameters based on Laplace transform technique by utilizing three sets of synchronized voltage and current phasors. Another approach based on synchronized voltage and current phasors obtained from both ends of the line is introduced in a recent paper, which emphasizes the benefits of online parameter estimation to distance relaying [15]. All these algorithms assume that highly precise synchronization is available and may yield significant errors if this does not hold.

Generally speaking, the existing online estimation algorithms perform well when the measurements utilized are accurate. In practice sometimes the measurements may contain errors due to various reasons like the current transformer saturation, data conversion errors or communication device abnormalities [16], [17]. Such measurement errors, as well as possible synchronization errors, could result in substantial inaccuracy in estimates.

Therefore, as pointed out in [18], it will be very desirable if an online parameter estimation approach can be designed capable of detecting and identifying measurement errors and synchronization inaccuracy. In this way, the bad measurements would be removed and only the sound measurements exploited to achieve a more precise estimate of the line parameters. Detection of synchronization inaccuracy can alert us to avoid synchronization assumption in parameter estimation and meanwhile take timely actions to remedy the problems leading to synchronization inaccuracy.

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Fig. 1. Transmission line considered for analysis.

This paper proposes an online algorithm for estimating the positive sequence line parameters able to make the most of all the measurements available and minimize the impacts of measurement and synchronization errors. The proposed solution is based on the fundamental frequency voltage and current phasors, which can be calculated from recorded waveforms or directly obtained from measuring devices such as phasor measurement units. We employ a two-terminal transmission line model for illustrating the solution.

The equivalent π circuit based on distributed parameter line model is harnessed to automatically and fully consider shunt capacitances and distributed parameter effects of long lines.

Section II illustrates the proposed method. The evaluation studies are reported in Section III, followed by the conclusion.

II. Proposed Optimal Estimator for Line Parameter Estimation

Section II-A presents the overall description of the proposed method, with elaborations given in Section II-B. Section II-C explains the procedure for detecting and identifying possible bad measurements.

A. Overall Description of the Method

Consider the line, assumed to be a transposed line, between terminals P and Q, as shown in Fig. 1, where E_G and E_H represent the Thevenin equivalent sources.

The proposed algorithm draws on the steady state voltage and current phasors at terminal P and Q that are measured at different moments such as every hour during normal operations. The aim is to estimate the positive sequence series resistance and reactance and shunt susceptance of the line per unit length.

Based on one set of voltage and current phasors at P and Q, two complex equations can be obtained based on the positive sequence equivalent π circuit, which link the phasors with the unknown line parameters. Separating the two complex equations into four real equations and solving these equations gives rise to the solution of unknown variables.

However, since the phasor estimates may contain errors, estimation of line parameters based on a single set of equations could be unreliable. Fortunately, a set of redundant equations may be derived by utilizing voltage and current measurements taken at different moments such as each hour [14]. Then, the nonlinear estimation theory may be utilized to obtain an optimal estimate of the line parameters and each of the phasors. Statistical approaches can also be adopted to detect, identify, and remove any possible bad data, and thus improve the estimation accuracy and increase the robustness of the algorithm by minimizing the impacts of possible measurement errors. More accurate estimates of the line parameters can lead to more accurate calculation of relay settings and more reliable operation of relaying algorithms.

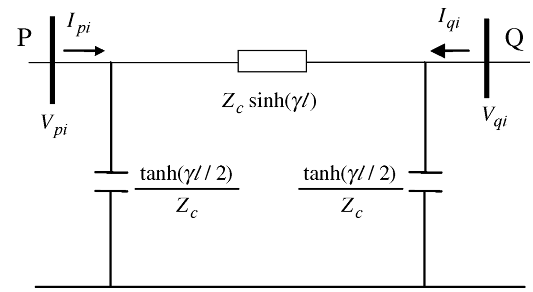

Fig. 2. Positive sequence network of the system during normal operation.

Note that zero-sequence components in the circuit during normal operations are usually negligible, thus the proposed algorithm mainly aims at estimating positive sequence line parameters instead of zero-sequence parameters. Nonetheless, if significant zero-sequence components arise due to various reasons such as unbalanced load conditions, existence of faults external to the studied line, etc., the proposed algorithm will be equally applicable for estimating zero-sequence parameters of the line.

The proposed approach for estimating positive sequence parameters is delineated in the following sections.

B. Proposed Optimal Estimator

The following discussion assumes that synchronized voltage and current measurements at P and Q during normal operation are available. Fig. 2 depicts the positive sequence network of the system during normal operation.

In the figure, the following notations are adopted.

V_{pi}, I_{pi}	i^{th} phasor measurement of positive sequence voltage and current at P;
V_{qi}, I_{qi}	i^{th} phasor measurement of positive sequence voltage and current at Q; $i = 1, 2, \ldots, N$, N being the total number of measurement sets, with each set consisting of V_{pi}, I_{pi}, V_{qi} and I_{qi};
Z_c	characteristic impedance of the line;
γ	propagation constant of the line;
l	length of the line in miles or kilometers (km).

Based on Fig. 2, we can derive the following equations [3], [7]:

$$V_{pi} - Z_c \sinh(\gamma l) I_{pi} + \sinh(\gamma l) \tanh(\gamma l/2) V_{pi} - V_{qi} e^{j\delta} = 0 \tag{1}$$

$$I_{pi} - \tanh(\gamma l/2) V_{pi}/Z_c + I_{qi} e^{j\delta} - \tanh(\gamma l/2) V_{qi} e^{j\delta}/Z_c = 0 \tag{2}$$

$$Z_c = \sqrt{z_1/y_1} \tag{3}$$

$$\gamma = \sqrt{z_1 y_1} \tag{4}$$

where

δ	the synchronization angle between measurements at P and Q, representing any possible synchronization error.
z_1, y_1	positive-sequence series impedance and shunt admittance of the line per mile or km, respectively.

The phasors in (1), (2) and synchronization angle are considered as known measurements. Define

$$M = [V_{p1}, I_{p1}, V_{q1}, I_{q1}, \ldots, V_{pN}, I_{pN}, V_{qN}, I_{qN}, \delta]. \quad (5)$$

δ will be assigned a value of zero since synchronized measurements are utilized. Modeling δ in the system equations could detect potential synchronization error due to synchronizing device failures, as shown in Section III. $M_i, i = 1, \ldots, (4N+1)$, designates the i^{th} element of M.

We define the measurement functions for each measurement as

$$Y_i(X) = x_{2i-1}e^{jx_{2i}}, \quad i = 1, \ldots, 4N \quad (6)$$

$$Y_{4N+1}(X) = x_{8N+1} \quad (7)$$

where X denotes the unknown variable vector, defined as

$$X = [x_1, x_2, \ldots, x_{8N}, x_{8N+1}, x_{8N+2}, x_{8N+3}, x_{8N+4}]^T \quad (8)$$

where

T	vector or matrix transpose operator;
$x_1, x_2, \ldots, x_{8N}$	variables required to represent the $4N$ complex measurements;
x_{8N+1}	synchronization angle;
x_{8N+2}, x_{8N+3} and x_{8N+4}	positive-sequence transmission line series resistance, series reactance and shunt susceptance per unit length, respectively.

By employing the defined variables, (1), (2) can be written as $f_{2i-1}(X) = 0$ and $f_{2i}(X) = 0$, respectively, as follows:

$$\begin{aligned} f_{2i-1}(X) = {} & x_{8i-7}e^{jx_{8i-6}} - Z_c \sinh(\gamma l) x_{8i-5}e^{jx_{8i-4}} \\ & + \sinh(\gamma l)\tanh(\gamma l/2) x_{8i-7}e^{jx_{8i-6}} \\ & - x_{8i-3}e^{jx_{8i-2}}e^{jx_{8N+1}} = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} f_{2i}(X) = {} & x_{8i-5}e^{jx_{8i-4}} - \tanh(\gamma l/2) x_{8i-7}e^{jx_{8i-6}}/Z_c \\ & + x_{8i-1}e^{jx_{8i}}e^{jx_{8N+1}} - \tanh(\gamma l/2) x_{8i-3} \\ & \times e^{jx_{8i-2}}e^{jx_{8N+1}}/Z_c = 0 \end{aligned} \quad (10)$$

where $i = 1, 2, \ldots, N$, representing the index of the measurement set

$$Z_c = \sqrt{(x_{8N+2} + jx_{8N+3})/(jx_{8N+4})} \quad (11)$$

$$\gamma = \sqrt{(x_{8N+2} + jx_{8N+3})(jx_{8N+4})} \quad (12).$$

Introduce S and $F(X)$ as the measurement vector and function vector, respectively, with their elements shown in the Appendix. The measurement vector and function vector are related by

$$S = F(X) + \mu \quad (13)$$

where, μ is determined according to meter characteristics.

The optimal estimate of X is obtained by minimizing the cost function defined as

$$J = [S - F(X)]^T R^{-1} [S - F(X)]. \quad (14)$$

The solution to (14) can be derived following the Newton-Raphson method [3]. After X is obtained, (6) can be applied to compute the estimated values of the measurement phasors.

For estimating positive-sequence line parameters, the proposed method is also applicable to complex topology such as parallel lines as long as the voltage and current measurements at two ends of the line are available, since there is no mutual coupling between positive-sequence circuits.

C. Detection and Identification of Bad Measurements

To detect the presence of bad measurement data, the method based on chi-square test as illustrated in [7] and [19] can be utilized. In this method, the expected value of the cost function is calculated first, which is equal to the number of degrees of freedom designated as k. Then the estimated value of the cost function C_J is obtained. If $C_J \geq \chi^2_{k,\alpha}$, then the presence of bad data is suspected with probability $(1-\alpha)$. Value of $\chi^2_{k,\alpha}$ can be calculated for a specific k and α based on chi-square distribution. If bad data exists, the measurement corresponding to the largest standardized error will be identified as the bad data. In our study, we choose α to be 0.01, indicating a 99% confidence level on the detection. More details are referred to [7, p. 655–664].

III. Case Studies

This section presents the case studies demonstrating the procedure and effectiveness of the proposed solution for detecting and identifying possible bad measurements and thus deriving a more accurate estimate of the line parameters.

A steady state analysis program has been developed to generate steady state phasors during normal operations. A 500-kV, 200-mile transmission line system with configuration as shown in Fig. 1 is modeled, with line parameters and source impedances being referred to [3]. The distributed parameter line model is used to calculate the equivalent series impedance and shunt admittance of the line, which is used in the steady state analysis program. Varying the angle difference between the voltage sources E_G and E_H will change the power flow transferred from terminal P to Q, and accordingly different sets of voltage and current phasors at P and Q can be produced. These phasors are then distorted with Gaussian noises with specified variances and then utilized as inputs to the developed algorithm for estimating the line parameters.

The algorithm has been implemented in Matlab. Representative results are reported in this section. The per unit system is utilized in the following discussions, with a base voltage of 500 kV and base voltampere of 1000 MVA. All the cases utilize

TABLE I
OPTIMAL ESTIMATES OF LINE PARAMETERS AND PHASORS WITHOUT BAD MEASUREMENTS

Quantity	Measured values	Optimal estimates
V_{p1} (p.u.)	0.82189 + j0.53537	0.82057 + j0.53457
I_{p1} (p.u.)	0.51781 + j0.7094	0.52166 + j0.71375
V_{q1} (p.u.)	0.97005 + j0.1433	0.96981 + j0.14534
I_{q1} (p.u.)	-0.65313 - j0.38769	-0.64951 - j0.38421
V_{p2} (p.u.)	0.85496 + j0.48538	0.85912 + j0.48732
I_{p2} (p.u.)	0.49938 + j0.65946	0.49677 + j0.65723
V_{q2} (p.u.)	0.99233 + j0.13169	0.98917 + j0.12903
I_{q2} (p.u.)	-0.60678 - j0.30728	-0.60846 - j0.30943
V_{p3} (p.u.)	0.92162 + j0.41448	0.91792 + j0.41406
I_{p3} (p.u.)	0.43283 + j0.55017	0.4338 + j0.54963
V_{q3} (p.u.)	0.9951 + j0.10816	0.99895 + j0.1091
I_{q3} (p.u.)	-0.53077 - j0.19337	-0.52993 - j0.19302
δ (degrees)	0	0.027705
line resistance (p.u./mile)	0.00099667	0.00094316
line reactance (p.u./mile)	0.0023566	0.0023762
line susceptance (p.u./mile)	0.0018349	0.0018315

three sets of measurements $(N = 3)$ and the following starting values: one for phasor magnitude, zero for phasor angle, zero for synchronization angle, 1E-3 for line resistance per unit length, 1E-3 for line reactance per unit length, and 1E-3 for line shunt susceptance per unit length. The estimator achieves the optimal estimate of line parameters quickly, around five iterations for all the cases. For the chi-square test, we choose α to be 0.01.

A. Cases Without Bad Measurements

This subsection studies the behavior of the algorithm when there are no bad measurements.

Assuming that all the measurements have the same error variance value of 1E-4, the optimal estimates of the line parameters and phasors are shown in Table I. The measured values and the optimal estimates are shown in the second and third column respectively. The line parameters in the second column indicate the actual line parameters.

We have $k = 9$, $\chi^2_{9,0.01} = 21.666$, and the estimated value of the cost function C_J is calculated as 2.3955. Since C_J is less than $\chi^2_{9,0.01}$, it is concluded based on Section II-C that no bad data exists.

It can be seen that quite accurate estimates have been achieved by the proposed method.

B. Cases With Bad Synchronization

Although synchronization based on global positioning system is normally highly precise, synchronization errors still may arise due to various reasons such as improper hardware wiring, unavailability of the GPS time reference and communication problems. This subsection illustrates how such errors may be pinpointed by the proposed method.

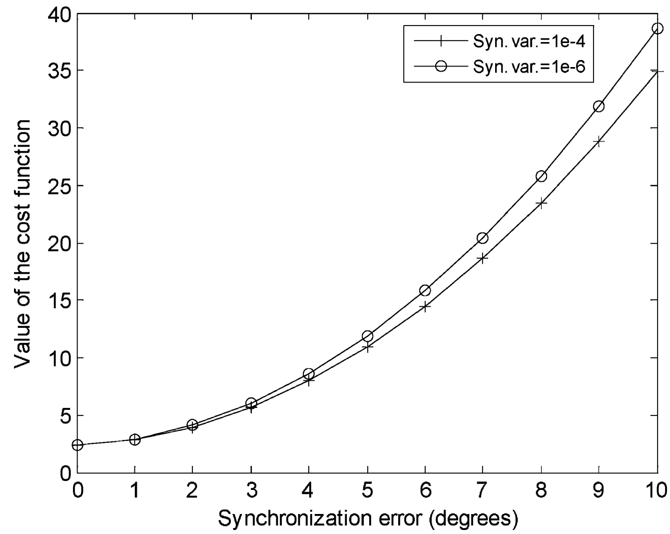

Fig. 3. Cost function versus the synchronization errors.

We first generate the measurement phasors. Next a synchronization error is applied to the measurements at terminal Q to emulate the synchronization error. Then the estimator is applied to obtain the results and compute the value of the cost function C_J. Fig. 3 depicts the calculated value of the cost function versus the synchronization error. One curve is obtained by using a value of 1E-6 for synchronization variance, and the other curve using 1E-4. Variances for other measurements are set to 1E-4. It can be seen that the cost function becomes considerably larger when the synchronization error reaches 6 degrees, which can be utilized to detect the presence of bad measurement data. As expected, Fig. 3 also reveals that a smaller variance value of the synchronization angle makes the estimator more sensitive to synchronization errors. A more detailed analysis is shown below.

Table II lists the optimal estimates when the synchronization has an error of 10 degrees using a value of 1E-6 for synchronization variance.

The estimated value of the cost function C_J is computed as 38.6139. Since C_J exceeds $\chi^2_{9,0.01}$, the presence of bad data is suspected, based on Section II-C.

To identify the possible bad data, the standardized errors are calculated and the largest standardized error is 6.0173, which corresponds to δ. Hence, δ is identified as the bad data. After removing δ from the measurement set, a new optimal estimate can be obtained as shown in Table III, which indicates that a more accurate estimate of the parameters has been reached. "N/A" means the corresponding measurement is not available. In this case, k is 8.0, $\chi^2_{8,0.01} = 20.09$, and the estimated value of the cost function C_J is 2.372. Since C_J is less than $\chi^2_{8,0.01}$, all the data are considered fairly accurate, and the estimates are regarded as acceptable.

Therefore, the optimal estimator is able to successfully detect and identify the possible synchronization errors, and obtain a more accurate estimate by employing only the reliable measurements.

TABLE II
OPTIMAL ESTIMATES OF LINE PARAMETERS AND PHASORS WITH BAD SYNCHRONIZATION

Quantity	Measured values	Optimal estimates
V_{p1} (p.u.)	0.82189 + j0.53537	0.81899 + j0.54556
I_{p1} (p.u.)	0.51781 + j0.7094	0.53716 + j0.71177
V_{q1} (p.u.)	0.9802 - j0.027324	0.98241 - j0.026031
I_{q1} (p.u.)	-0.71053 - j0.26839	-0.68662 - j0.26274
V_{p2} (p.u.)	0.85496 + j0.48538	0.85777 + j0.49139
I_{p2} (p.u.)	0.49938 + j0.65946	0.51108 + j0.6618
V_{q2} (p.u.)	1.0001 - j0.042627	0.99751 - j0.040914
I_{q2} (p.u.)	-0.65092 - j0.19724	-0.63507 - j0.19519
V_{p3} (p.u.)	0.92162 + j0.41448	0.91694 + j0.40859
I_{p3} (p.u.)	0.43283 + j0.55017	0.4489 + j0.56397
V_{q3} (p.u.)	0.99877 - j0.066278	0.99817 - j0.052813
I_{q3} (p.u.)	-0.55628 - j0.098266	-0.54378 - j0.093658
δ (degrees)	0	0.011675
line resistance (p.u./mile)	0.00099667	0.0014293
line reactance (p.u./mile)	0.0023566	0.0034997
line susceptance (p.u./mile)	0.0018349	0.0024303

TABLE III
OPTIMAL ESTIMATES OF LINE PARAMETERS AND PHASORS AFTER SYNCHRONIZATION ANGLE DATA IS REMOVED

Quantity	Measured values	Optimal estimates
V_{p1} (p.u.)	0.82189 + j0.53537	0.82063 + j0.53433
I_{p1} (p.u.)	0.51781 + j0.7094	0.52125 + j0.71374
V_{q1} (p.u.)	0.9802 - j0.027324	0.98027 - j0.02521
I_{q1} (p.u.)	-0.71053 - j0.26839	-0.70684 - j0.26561
V_{p2} (p.u.)	0.85496 + j0.48538	0.85915 + j0.48725
I_{p2} (p.u.)	0.49938 + j0.65946	0.49642 + j0.65708
V_{q2} (p.u.)	1.0001 - j0.042627	0.99652 - j0.044755
I_{q2} (p.u.)	-0.65092 - j0.19724	-0.65339 - j0.19914
V_{p3} (p.u.)	0.92162 + j0.41448	0.9179 + j0.41423
I_{p3} (p.u.)	0.43283 + j0.55017	0.43348 + j0.54927
V_{q3} (p.u.)	0.99877 - j0.066278	1.0028 - j0.066347
I_{q3} (p.u.)	-0.55628 - j0.098266	-0.55571 - j0.098164
δ (degrees)	N/A	10.2775
line resistance (p.u./mile)	0.00099667	0.00093137
line reactance (p.u./mile)	0.0023566	0.0023485
line susceptance (p.u./mile)	0.0018349	0.0018164

TABLE IV
OPTIMAL ESTIMATES OF LINE PARAMETERS AND PHASORS WITH BAD VOLTAGE MEASUREMENT

Quantity	Measured values	Optimal estimates
V_{p1} (p.u.)	0.98627 + j0.64245	0.95422 + j0.62266
I_{p1} (p.u.)	0.51781 + j0.7094	0.52723 + j0.71686
V_{q1} (p.u.)	0.97005 + j0.1433	0.99817 + j0.16352
I_{q1} (p.u.)	-0.65313 - j0.38769	-0.65767 - j0.379
V_{p2} (p.u.)	0.85496 + j0.48538	0.87891 + j0.49956
I_{p2} (p.u.)	0.49938 + j0.65946	0.4919 + j0.65531
V_{q2} (p.u.)	0.99233 + j0.13169	0.97071 + j0.11704
I_{q2} (p.u.)	-0.60678 - j0.30728	-0.60473 - j0.31174
V_{p3} (p.u.)	0.92162 + j0.41448	0.93511 + j0.42323
I_{p3} (p.u.)	0.43283 + j0.55017	0.43057 + j0.54731
V_{q3} (p.u.)	0.9951 + j0.10816	0.98324 + j0.10014
I_{q3} (p.u.)	-0.53077 - j0.19337	-0.52722 - j0.1947
δ (degrees)	0	-0.0024721
line resistance (p.u./mile)	0.00099667	0.0014317
line reactance (p.u./mile)	0.0023566	0.0023126
line susceptance (p.u./mile)	0.0018349	0.0017765

C. Cases With Bad Voltage or Current Measurements

Large errors in voltage or current measurements can lead to considerable inaccuracy in parameter estimates. This subsection illustrates how such bad measurements can be detected by the optimal estimator. A value of 1E-6 for synchronization variance and 1E-4 for other measurements are utilized.

Suppose that there is an error of 20% in the magnitude of V_{p1}, then the optimal estimates will be obtained as shown in Table IV.

The estimated value of the cost function C_J is calculated as 79.6378, which is larger than $\chi^2_{9,0.01} = 21.666$. Therefore, presence of bad measurements is suspected and V_{p1} is successfully identified as the bad data.

After the bad measurement is removed, a new set of optimal estimates are calculated as shown in Table V. In this case, $k = 7$, $\chi^2_{7,0.01} = 18.475$, and the estimated value of the cost function C_J is 2.2706. Since C_J is less than $\chi^2_{7,0.01}$, all the data are considered fairly accurate and the estimates are regarded as satisfactory. Comparison between Tables IV and V manifests that the line parameter estimation accuracy is considerably enhanced.

Similarly, bad current measurements may also be successfully detected and identified. Suppose that there is an error of 20% in the magnitude of I_{p1}, then the estimates will be derived as shown in Table VI. The estimated value of the cost function C_J is calculated as 81.312, which is greater than $\chi^2_{9,0.01} = 21.666$. Therefore, presence of bad measurements is suspected and I_{p1} is identified as bad data.

After the bad current measurement is removed, a new set of optimal estimates are calculated as shown in Table VII. In this case, $k = 7$, $\chi^2_{7,0.01} = 18.475$, and the estimated value of the

TABLE V
OPTIMAL ESTIMATES OF LINE PARAMETER AND PHASORS WITH BAD VOLTAGE MEASUREMENT BEING REMOVED

Quantity	Measured values	Optimal estimates
V_{p1} (p.u.)	N/A	0.81473 + j0.53124
I_{p1} (p.u.)	0.51781 + j0.7094	0.52155 + j0.71368
V_{q1} (p.u.)	0.97005 + j0.1433	0.96858 + j0.14463
I_{q1} (p.u.)	-0.65313 - j0.38769	-0.64918 - j0.38447
V_{p2} (p.u.)	0.85496 + j0.48538	0.85825 + j0.48687
I_{p2} (p.u.)	0.49938 + j0.65946	0.49695 + j0.65726
V_{q2} (p.u.)	0.99233 + j0.13169	0.98999 + j0.12948
I_{q2} (p.u.)	-0.60678 - j0.30728	-0.60854 - j0.30929
V_{p3} (p.u.)	0.92162 + j0.41448	0.91717 + j0.4137
I_{p3} (p.u.)	0.43283 + j0.55017	0.43393 + j0.54972
V_{q3} (p.u.)	0.9951 + j0.10816	0.99962 + j0.10946
I_{q3} (p.u.)	-0.53077 - j0.19337	-0.52998 - j0.19291
δ (degrees)	0	0.00038015
line resistance (p.u./mile)	0.00099667	0.00092429
line reactance (p.u./mile)	0.0023566	0.0023836
line susceptance (p.u./mile)	0.0018349	0.0018355

TABLE VI
OPTIMAL ESTIMATES OF LINE PARAMETERS AND PHASORS WITH BAD CURRENT MEASUREMENT

Quantity	Measured values	Optimal estimates
V_{p1} (p.u.)	0.82189 + j0.53537	0.82374 + j0.53603
I_{p1} (p.u.)	0.62137 + j0.85128	0.58756 + j0.81357
V_{q1} (p.u.)	0.97005 + j0.1433	0.98272 + j0.1302
I_{q1} (p.u.)	-0.65313 - j0.38769	-0.68713 - j0.41756
V_{p2} (p.u.)	0.85496 + j0.48538	0.86043 + j0.48057
I_{p2} (p.u.)	0.49938 + j0.65946	0.49637 + j0.66786
V_{q2} (p.u.)	0.99233 + j0.13169	0.98345 + j0.13429
I_{q2} (p.u.)	-0.60678 - j0.30728	-0.61089 - j0.30494
V_{p3} (p.u.)	0.92162 + j0.41448	0.91818 + j0.4081
I_{p3} (p.u.)	0.43283 + j0.55017	0.43481 + j0.56013
V_{q3} (p.u.)	0.9951 + j0.10816	0.99361 + j0.11324
I_{q3} (p.u.)	-0.53077 - j0.19337	-0.53226 - j0.18952
δ (degrees)	0	-0.0049759
line resistance (p.u./mile)	0.00099667	0.00090433
line reactance (p.u./mile)	0.0023566	0.002237
line susceptance (p.u./mile)	0.0018349	0.0019697

TABLE VII
OPTIMAL ESTIMATES OF LINE PARAMETER AND PHASORS WITH BAD CURRENT MEASUREMENT BEING REMOVED

Quantity	Measured values	Optimal estimates
V_{p1} (p.u.)	0.82189 + j0.53537	0.82081 + j0.53486
I_{p1} (p.u.)	N/A	0.52873 + j0.72348
V_{q1} (p.u.)	0.97005 + j0.1433	0.97106 + j0.14371
I_{q1} (p.u.)	-0.65313 - j0.38769	-0.65343 - j0.38749
V_{p2} (p.u.)	0.85496 + j0.48538	0.85926 + j0.48665
I_{p2} (p.u.)	0.49938 + j0.65946	0.49679 + j0.65825
V_{q2} (p.u.)	0.99233 + j0.13169	0.98863 + j0.12958
I_{q2} (p.u.)	-0.60678 - j0.30728	-0.60866 - j0.309
V_{p3} (p.u.)	0.92162 + j0.41448	0.91796 + j0.41344
I_{p3} (p.u.)	0.43283 + j0.55017	0.43395 + j0.55067
V_{q3} (p.u.)	0.9951 + j0.10816	0.99844 + j0.10957
I_{q3} (p.u.)	-0.53077 - j0.19337	-0.53012 - j0.19268
δ (degrees)	0	-0.00028832
line resistance (p.u./mile)	0.00099667	0.00094006
line reactance (p.u./mile)	0.0023566	0.0023649
line susceptance (p.u./mile)	0.0018349	0.001846

cost function C_J is 1.3604. Since C_J is less than $\chi^2_{7,0.01}$, all the data are considered fairly accurate and the estimates are regarded as satisfactory. Comparing Tables VI and VII evidences that the line parameter estimation accuracy is significantly improved.

IV. CONCLUSION

This paper presents an algorithm for estimating the positive sequence parameters of a transmission line by utilizing online voltage and current phasors measured at different moments from two terminals of the line during normal operations. This paper demonstrates that it may be feasible to design an approach for detecting, identifying and removing possible bad measurements and thus improving the estimation accuracy. When synchronized measurements are employed, possible synchronization errors can also be detected, thus enhancing the line parameter estimation accuracy. The developed algorithm is based on distributed parameter line model and thus fully considers the effects of shunt capacitance and distributed parameter effects of long lines. Quite encouraging results have been obtained by simulation studies.

APPENDIX

Elements of vector S are

$$S_i = 0, \quad i = 1, \ldots, 4N \tag{A.1}$$

$$S_{2i+4N-1} = abs(M_i), \quad i = 1, 2, \ldots, 4N \tag{A.2}$$

$$S_{2i+4N} = angle(M_i), \quad i = 1, 2, \ldots, 4N \tag{A.3}$$

$$S_{12N+1} = M_{4N+1} \tag{A.4}$$

where $abs(.)$ and $angle(.)$ yield the magnitude and angle of the input argument, respectively.

Elements of function vector $F(X)$ are

$$F_{2i-1}(X)=\mathrm{Re}\left(f_i(X)\right),\ i=1,\ldots,2N \tag{A.5}$$

$$F_{2i}(X)=\mathrm{Im}\left(f_i(X)\right),\ i=1,\ldots,2N \tag{A.6}$$

$$F_{2i+4N-1}(X)=abs\left(Y_i(X)\right)=x_{2i-1},\ i=1,\ldots,4N \tag{A.7}$$

$$F_{2i+4N}(X)=angle(Y_i(X)=x_{2i},\ i=1,\ldots,4N \tag{A.8}$$

$$F_{12N+1}(X)=Y_{4N+1}(X)=x_{8N+1}. \tag{A.9}$$

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