A Power Waveform Classification Method for Adaptive Synchrophasor Estimation

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Abstract—This paper designed a novel multiresolution analysis method and input waveform classification scheme. The proposed techniques serve as preprocessing procedures of input waveforms in order to facilitate synchrophasor estimation. Simple, scalable and time-shifted “pseudowavelets” (PWs) are employed to repeatedly calculate the correlation coefficients between the proposed PWs and input power waveforms. By scrutinizing the correlation factors with respect to frequency and time, the proposed method is capable of revealing the temporal trajectories of frequency and amplitude features of input waveforms. Such features are then leveraged to classify input waveform types. The result of such waveform classification can be applied to perform accurate synchrophasor estimation. Contrary to traditional approaches, where a single algorithm is designed to compute synchrophasor accurately for all input signal types, in this paper, a framework is proposed which enables adaptive switching of synchrophasor algorithms, so that the most suitable algorithm can be used for the identified waveform. The efficacy and efficiency of proposed methods are validated with standardized phasor measurement unit testing waveforms and simulated power system waveforms. The results prove that the PW-based waveform classification method is capable of distinguishing between power system dynamic waveforms.

Index Terms—Correlation, frequency estimation, phasor measurement unit (PMU) calibration, power system measurements, pseudowavelets (PWs), spectral analysis, synchronized phasor measurements, wavelet transform (WT).

I. INTRODUCTION

SYNCHROPHASOR measurement technology (SMT) is empowered by the increasing deployment of phasor measurement units (PMUs) and intelligent electronic devices (IEDs) with PMU functions. Over the years, SMT has proven to be a good investment and promising enhancement in many power grid applications [1]–[3].

A healthy operation of synchrophasor system is guaranteed by a life-cycle management of the system [4]. Synchrophasor data sources, i.e., PMUs and IEDs, are calibrated before deployment, and maintained periodically during service in the power grid. Either effort necessitates a highly accurate reference to facilitate the calibration process.

The development of PMU calibration systems has been an ongoing effort for a decade to facilitate acceptance test for PMUs before their deployment in power grids. In this procedure, test waveforms are quantified and standardized [5], [6]. So far, many organizations have developed PMU calibration labs in independent efforts. In [7] and [8], an accurate signal generator is used in the lab, and synchrophasor reference is thereby inferred from the settings of the signal generator, which are known and determined before the tests. Practically, the credibility of such PMU calibration systems depends solely on the accuracy of the waveform generation system. Since waveform generation procedure is subject to uncertainties such as noise and hardware delay, as a result, those uncertainties should be effectively estimated and compensated [9].

A common goal found in the literature is to utilize an absolutely accurate algorithm applicable to all dynamic conditions. Since the electric power system operating conditions are constantly evolving and exhibiting new phenomena, this “one-size-fits-all” approach is becoming increasingly impractical. Research on synchrophasor estimation started from over three decades ago, when the discrete Fourier transform (DFT) method was proposed [10]. Over the years, synchrophasor estimation became an interdisciplinary topic where a wide spectrum of signal processing methods was employed. The two most widely used strategies are to explore signal properties in the frequency and time domains [8], [11]. The difference between the two strategies is in the selection of signal models, and in the approach how to compensate for model imperfections. DFT uses static signal model, and consequently, frequency leakage needs to be compensated [12]. Thanks to the frequency-selection feature of DFT, frequency-domain methods work well for signals at off-nominal frequencies and/or when infiltrated with harmonics. Since synchrophasor was redefined as a dynamic quantity in [13], more research efforts were directed to dynamic synchrophasor estimation [14], which is frequently done in time domain where curve-fitting techniques are extensively used [11], [13]. Time-domain methods are suitable for processes with slow transients, such as modulation, but may incur large errors with the presence of abrupt changes, such as harmonics. As a result, variations benefitting from both approaches have been proposed [15]. However, to the best of the authors’ knowledge, a single...
The waveforms in the electric power grid follow specific patterns, and they have been generalized and formulated as PMU laboratory test waveforms in [5]. As a result, each PMU test waveform is associated with certain electrical phenomenon in the real power grid [16]. For example, low-frequency oscillations between areas will cause the waveforms to become amplitude modulated; when a generator gradually loses synchronism, its frequency will ramp up or down (frequency chirp). For each type of waveform, there are mature methods in the power grid research [8], [11]–[20]; or can be borrowed from other disciplines where it has been extensively studied, for instance, amplitude-modulated signals [21], frequency-modulated signals [21]–[23], and frequency chirp [24]–[26].

Acknowledging this, this paper proposed an input waveform classification method, which can then be leveraged to select the most suitable algorithm for accurate synchrophasor estimation. Section II elaborates the problem of interest. The mathematical derivation of a novel time–frequency multiresolution analysis method is discussed in Section III. In Section IV, a waveform classification method utilizing time–frequency analysis result is described. Tests on implementation platforms are performed and analyzed in Section V. Conclusions are listed in Section VI.

II. PROBLEM DESCRIPTION

A. Proposed Approach for Synchrophasor Estimation

The framework is shown in Fig. 1. Waveform classification and synchrophasor estimation are encapsulated in two decoupled loops. The proposed waveform classification method [Loop, Fig. 1(a)] constantly retrieves the input samples from data buffer, and saves the most recent classification result in a register. Upon each synchrophasor estimation [Loop, Fig. 1(b)], the latest waveform classification result is retrieved from the register, and algorithms can be selected adaptively for synchrophasor estimation. With identified waveform type, the algorithm specifically designed for that type of waveform can be selected. Since such algorithm does not need to accommodate multiple types of waveforms, it is also expected to have simpler mathematical structures. Overall, more accurate results and less computation time can be achieved.

Note that in Fig. 1, Loop [Fig. 1(a)] and Loop [Fig. 1(b)] run independently and do not need to be time synchronized. The maximum computation time of Loop [Fig. 1(b)] is determined by the selected reporting rate per IEEE standard. Loop [Fig. 1(a)] can operate at a different pace which may be reasonably slower. It is justifiable to decouple waveform classification and synchrophasor estimation loops, since the power grid inertia determines that waveform type does not change in a short period of time [27]. As a result, both accuracy and efficiency can be achieved with the proposed strategy. Also, the procedure of creating output synchrophasor stream is simplified in Fig. 1.

B. Feature Extraction and Classification

The key of the proposed waveform classification effort lies in the extraction of distinct features of input waveform. Time–frequency signatures are good candidates since approaches to extracting such quantities in signal processing/machine learning are relatively mature [28]. Since power system signals evolve in both time and frequency, short-time Fourier transform (STFT) and wavelet transform (WT) may be utilized. The major pitfall of STFT is invariable time–frequency atom (resolution), and consequently, it usually cannot provide enough details on either the time or frequency features. On the contrary, in WT, scalable and time-shifted versions of mother wavelet, termed “children wavelets,” are used. By repeatedly scanning the input waveform with children wavelets, the correlation between child wavelet and truncated input signal can be quantified for each time–frequency atom. In WT, the scaling factor of mother wavelet is used as a measure of the variations in input signal, and is considered a generalized interpretation of frequency. In the power grid, WT is frequently used in power quality assessment issues [29], [30], as well as in low-frequency electromechanical oscillation studies [31], and disturbance analysis [32]. There are multiple mother wavelet families and members, and consequently, using different mother wavelets will yield distinct results, specifically the interpretation of scales, shown in Fig. 2.
Moreover, as illustrated in Fig. 3, continuous wavelet transform (CWT) may reveal certain features of input waveform. For instance, harmonic component can be seen in Fig. 3(b) as a separate spectrum (pointed by arrow), and correlation strength variation can be observed in amplitude modulation in Fig. 3(c). However, even with 12-cycle data window, the time–frequency features among different waveforms lack enough contrast from each other to enable effective waveform classification.

Inspired by WT, this paper proposed an alternative multiresolution tool to extract the time–frequency features of input signal, which are then used to classify input signal types. Instead of mother wavelets, a set of customized “pseudowavelets (PWs)” is used in order to fully employ the trigonometric nature of input power waveforms.

III. MULTRESOLUTION TIME–FREQUENCY ANALYSIS BASED ON PSEUDOWAVELETS

A multiresolution analysis is performed on input power waveforms using the proposed PW method. In the procedure, input waveform is translated to coefficient factors with respect to a pair of time and PW frequency. The coefficients are then leveraged to reveal the trajectory of frequency, and amplitude (energy) along time, which are considered as “signatures” of an input waveform.

A. Power Waveform Revisited

In power systems, the instant power quantities at a certain node are given by the superposition of the contributions from generators at the node in question [16]. In general, this superposition depends on the relative electrical distance between the node and power generators. As a result, the basic power signal model is shown in the following equation [5]:

\[ x(t) = a(t) \cdot \cos \left[ 2\pi \int f(t)dt + \phi_0 \right] \]  

where \( a(t) \) is the instant amplitude, \( f(t) \) is the instant frequency, and \( \phi_0 \) is initial phase angle. It should be pointed out that the definition of “instant” amplitude and frequency is still under debate in academia [33]. Regardless, without further discussion, model (1) is utilized as a general expression of input signal.

Depending on the assignments of \( a(t) \) and \( f(t) \), (1) may denote signals under various operating conditions, which are elaborated in the related IEEE standard [5].

B. Continuous Wavelet Transform and Proposed “Pseudowavelet”-Based Method

CWT is defined in (2)

\[ \text{CWT}(x, a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) \psi^*_a, b \left( \frac{t - b}{a} \right) dt \]  

where \( \psi(t) \) is the mother wavelet function, \( a \) denotes the scaling factor, \( b \) represents the time shift, and * is complex conjugate. Functions \( \psi_{a,b}(t) := 1/\sqrt{|a|} \cdot \psi[(t - b)/a] \) are named “children wavelet” [34].

Mathematically, CWT computes the correlation factors between input \( x(t) \) and children wavelet \( \psi_{a,b}(t) \), characterized by pairs of \( a \) and \( b \) values. Since wavelets have finite time support (time limited), they also serve as windows that crop input waveform, resulting in “hopping” windows similar to STFT. The application of “repeated scanning” on a multiscale level enables CWT, the capability to reveal both long-term trends and short-term fluctuations in an input waveform. Evidently, mother wavelet should be meticulously selected so that CWT can yield the most meaningful results, and thus different families/members of mother wavelets are applied in various fields. Such examples may be found in chemometrics [35], hydrology [36], and power waveform quality [37]. In the context of power grid signals, model (1) should be leveraged as prior information, and this is the main motivation of the PW method.

Similar to CWT, we define a PW analysis, shown in the following equation:

\[ \gamma(x; a, b) = \int_{-\infty}^{\infty} \tilde{x}(t) \vartheta \left( \frac{t - b}{a} \right) dt \]

where \( \gamma(x; a, b) \) is the correlation coefficient between \( \tilde{x}(t) \) and \( \vartheta(t) \) for selected pair of \( a \) and \( b \), \( \tilde{x}(t) := [x(t) - \bar{x}] / \max[x(t)] \) denotes the detrended and normalized input signal, and \( \vartheta(t) \) is the proposed PWs, which are unit-amplitude cosine waves. Also, only real signals are analyzed, and therefore, complex conjugate in (2) is not applied.

C. Extracting Time–Frequency Information Using Pseudowavelets

Similar to CWT, the proposed method essentially performs correlation calculations on a multiresolution level, and therefore is also a redundant transform [34]. Since this paper only focuses on revealing signal composition, and therefore, strict mathematical discussions on the “transform” and “inverse transform” are avoided.
According to Fourier theory, $x(t)$ can be decomposed into an infinite number of sinusoidal waves, shown in the following equation:

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df.$$  \hspace{1cm} (4)

For simplicity, only one single-frequency component $x_1(t)$ is analyzed. Also, we denote the detrended and normalized input signal as $x(t)$ from now on. In the derivations, consider the integral (3) between $x_1(t)$ and a PW $X_{pw}(f)$ defined at frequency $f = f_{pw}$, where $f_{pw}$ is in fact determined by the scaling parameter $a$. Furthermore, considering that the PW is time limited and only has values over an interval of $T_{pw} := 1/f_{pw}$, it is appropriate in the discussion that we force the beginning of PW to be $\tau = 0$, therefore omitting the time-shifting parameter $b$ all together. Instead, the initial phase angle $\phi_1$ is treated as changeable as PW moves along input signal. Therefore, a simpler expression of (3) can be shown in the following equation:

$$\gamma(\tau, f_{pw}) = \int_{0}^{T_{pw}} x_1(t + \tau) \phi(t, f_{pw}) dt$$ \hspace{1cm} (5)

where $x_1(t) = \cos(2\pi f_1 t + \phi_1)$ is the input with unknown frequency $f_1$ and $\phi(t, f_{pw}) := \cos(2\pi f_{pw} t)$. $\tau$ is time lag, and it only changes the instant phase angle of $x_1(t)$.

If we rewrite (5), there is

$$\gamma(o_1, \tau) = \int_{0}^{T_{pw}} \cos(o_1 t + o_1 \tau + \phi_1) \cos(o_{pw} t) dt$$ \hspace{1cm} (6)

where $o_1 := 2\pi f_1$ and $o_{pw} := 2\pi f_{pw}$. Denote $\phi'_1 := o_1 \tau + \phi_1$. Using trigonometric properties, (5) can be broken down as

$$\gamma(o_1, \phi'_1) = \frac{1}{2} \int_{0}^{T_{pw}} \cos[(o_1 + o_{pw}) t + \phi'_1] dt + \frac{1}{2} \int_{0}^{T_{pw}} \cos[(o_1 - o_{pw}) t + \phi'_1] dt.$$  \hspace{1cm} (7)

Depending on the value of $(o_1 - o_{pw})$, the evaluation of (6) is discussed separately as follows:

**Scenario 1:** $o_1 - o_{pw} = 0$:

$$\gamma(o_1, \tau) = \frac{1}{2} \int_{0}^{T_{pw}} \cos(2o_1 t + \phi'_1) dt + \frac{1}{2} \int_{0}^{T_{pw}} \cos(\phi'_1) dt = \frac{T_{pw}}{2} \cos\phi'_1 = \frac{\pi}{o_{pw}} \cos(o_{pw} \tau + \phi_1).$$  \hspace{1cm} (8)

The first integral is zero since $2o_1 = 2o_{pw}$, and the integration on the second harmonic over a period is zero.

**Scenario 2:** $o_1 - o_{pw} \neq 0$:

$$\gamma = \frac{o_1}{2(o_1 + o_{pw})} \sin(o_1 T_{pw} + \phi'_1) - \sin\phi'_1$$

$$+ \frac{1}{2(o_1 - o_{pw})} \sin(o_1 T_{pw} + \phi'_1) - \sin\phi'_1$$

$$= \frac{\sin(o_1 T_{pw} + \phi'_1) - \sin\phi'_1}{o_1^2 - o_{pw}^2}.$$  \hspace{1cm} (9)

**D. Further Discussion on $\gamma(\tau)$**

Applying L’Hospital’s rule, the limit of (9) as $o_1 \to o_{pw}$ can be evaluated. Note that $o_{pw} T_{pw} \equiv 2\pi$

$$\lim_{o_1 \to o_{pw}} \gamma = \lim_{o_1 \to o_{pw}} \left[ \sin(o_1 T_{pw} + \phi'_1) - \sin\phi'_1 \right] o_1$$

$$\frac{\sin(o_1 T_{pw} + \phi'_1) - \sin\phi'_1}{2o_1}$$

$$+ \lim_{o_1 \to o_{pw}} \frac{\sin(o_1 T_{pw} + \phi'_1) - \sin\phi'_1}{2o_1}$$

$$= \frac{T_{pw}}{2} \cos\phi'_1$$

$$= \text{Right-hand side of (8)}. \hspace{1cm} (10)$$

To conclude, the evaluation of (5) can be compactly expressed using (10), when the limit value at $o_1 = o_{pw}$ is specified. The zeros of (8) are acquired by solving

$$[\sin(o_1 T_{pw} + \phi'_1) - \sin\phi'_1] o_1 = 0, o_1 \neq o_{pw}$$

$$\omega_1 = k o_{pw}, \hspace{0.5cm} k = 0, 2, 3, 4, \ldots$$

$$o_{pw} \geq o_{pw,\min} \equiv \frac{2\pi}{T_{window}}, \hspace{1cm} (11a)$$

$$o_1 = \frac{(2k + 1)\pi - 2\phi_1}{T_{pw} + 2\tau}, \hspace{0.5cm} k = 0, 1, 2, 3, \ldots \hspace{1cm} (11b)$$

where $T_{window}$ is the length of data observation window in seconds. $o_{pw,\min}$ corresponds to the situation where only one cycle of cosine wave spans the entire window length.

From (11a), the evaluation of integral (5) will be zero when the PW frequency value $o_{pw}$ is zero (trivial case), or the integer fractions of the actual (unknown) signal frequency. Due to this behavior of correlation intensity $\gamma(\tau)$, when tracking the zeros of $\gamma(\tau)$, the focus should be on the frequencies around the integer fractions of 60 Hz. For instance, Fig. 4 shows the correlation intensity at lag $\tau = 0.01$ s. When the input waveform is a steady sinusoidal wave at 64 Hz, correlation intensities will be zero at frequencies 32, 21.33, 16, 12.8, 10.67, 9.14, 8, 7.11, and 6.4 Hz. In this case, since ten nominal
cycles of data are used, the minimal PW frequency is 6 Hz. The scale of frequency axis is adjusted to show details at lower frequencies.

Moving $\gamma(t)$ along time axis while computing the correlation intensity using (5) with respect to all the PW frequency component, the values of $\gamma(t,\omega_{pw})$ are calculated. A matrix $\Gamma$ can be thereafter formed, with its elements being $\gamma(t,\omega_{pw})$.

Fig. 5 shows a simple illustration for the aforementioned example, with time lag from 0 to 0.16 s. Geometrically, given input signal, $\Gamma$ is visualized as a surface, with $x$-axis being time lag $t$, $y$-axis being PW frequency $\omega_{pw}$, and $z$-axis being the value of $\gamma(t,\omega_{pw})$. When projected onto $t - \omega_{pw}$ plain, correlation intensity, the absolute value of $\Gamma$, can be depicted as brightness in Fig. 5(a) and (b), or contours in Fig. 5(c) and (d). As can be seen, the regions of low values of $\Gamma$ form distinct “zero bands,” indicated by red-dotted lines.

It is worth mentioning that the proposed method is not designed for accurate frequency estimation, since the correlation integral is merely an approximation, and its accuracy depends on the selected sampling interval and data window length. However, the accuracy of such calculation is sufficient for the purpose of input waveform classification. In obtaining the zero-band frequencies, (11a) is a sufficient, but not necessary condition. As shown in Fig. 4, $\gamma(0.01)$ may achieve zero values at frequencies other than the integer fractions of 64 Hz.

E. Matrix Formulation of Pseudowavelet Method

Intuitively, the time–frequency analysis method in Sections III-C and III-D involves iterations on both frequency and time. To expedite calculation, matrix formulation is explored.

Similar to STFT, the proposed PW method can be considered as performing repeated calculation with hopping windows along data array. Such procedure is illustrated in Fig. 6. Meaningful data are depicted as greyed rectangles, forming input signal matrix $X_{\text{input}}$. To enable matrix multiplication, truncated data arrays are zero padded to maintain the same lengths. The initialization of $X_{\text{input}}$ is shown in Algorithm 1.

Algorithm 1 Initialization of matrix $X_{\text{input}}$ (hop size $= 1$, array size $= 4$)

1. READ input $x(t)$
2. DETREND & NORMALIZE input $x(t)$, yielding $\hat{x}(t)$
3. CIRCULAR SHIFT $\hat{x}(t)$ and populate into matrix $X_{N \times N}$
4. CALCULATE upper triangular matrix of $X_{N \times N}$
5. FLIP along $X_{\text{input}}$ the center, and result in input signal matrix $X_{\text{input}}$

The transformation matrix comprises PWs (cosine waves) of a set of customized frequencies, and should be generated offline. Since each PW has finite support, truncation of input signal is also conducted when PW row vector multiplies input signal column vector. The values of maximum frequency of PW $f_{pw,\text{max}}$ as well as frequency resolution $\Delta f_{pw}$ are chosen, so that enough frequency details can be provided. The value of minimum frequency of PW $f_{pw,\text{min}}$ is determined by the reciprocal of total data length, which is incidentally the frequency resolution of Fourier methods.

IV. WAVEFORM CLASSIFICATION TO IMPROVE SYNCHROPHASOR ESTIMATION

Section III introduces a novel approach to extract time-frequency information from input signal. For the purpose of input waveform classification, it is of paramount importance that this information is leveraged to highlight energy (amplitude) and/or frequency features. The term “feature” in this paper denotes the quantity that can present unique behaviors in a certain phenomenon, similar to that in machine learning [38]. Since the power grid is dynamic system, it is intuitive to select features that signify such nature, for instance, frequency patterns and amplitude patterns. After input waveform type is identified, suitable algorithms may be applied, respectively. In this paper, a simple reference algorithm is proposed.

An overview of the proposed waveform classification method is illustrated in Fig. 7. “Data conditioning”
includes data truncation, downsampling, detrending, and normalizing. Details of each procedure are discussed in Sections IV-A and IV-B.

A. Frequency Trajectory Extraction

As is manifested in Fig. 5, the key is not only to extract instant frequency values, but more importantly, the trajectory of frequency along time. Bear in mind that Fig. 5 is in fact a visualization of matrix $\Gamma$, with columns correspond to time instants ($x$-axis), rows represent PW frequencies ($y$-axis), and matrix elements denotes correlation intensities ($z$-axis), which are depicted in color patterns. As discussed in Section III, frequency patterns can be traced by tracking the zero bands in matrix $\Gamma$ elements: $\Gamma_{i,j} := \gamma_{ij}$. By doing so, $z$-axis is reduced (since correlation intensity is assumed to be zero), and progression of frequency along time can be extracted.

With a closer observation of the contours, it can be seen that the most prominent frequency and time information can be tracked from the points at which the contours have zero derivatives with respect to time ($x$-axis). In practice, contour $\gamma_{ij} = \text{constant}$ may not be drawn since the calculated matrix elements do not necessarily contain the desired constant value. Rather, a “region” in the vicinities of the specified constant value should be considered. Moreover, the “discontinuity” of matrix $\Gamma$ elements makes it tricky to directly evaluate contour derivatives using the differentials of matrix elements. Therefore, a method evaluating the “occurrence rate spectrum” of each frequency component is adopted, and elaborated as follows.

1) Specify threshold and tolerance for a contour region, for example, $\gamma_{ij} = (20 \pm 8) \times 10^{-5}$. The elements falling into this range will all be considered as one contour.
2) Extract the elements within the contour, and sort elements by their frequency ($y$-axis) and time ($x$-axis) values.
3) The frequency indices with the highest densities (frequency occurrence rate, FOR) correspond to the “flattest” portions of contour, and thus are the zero-band frequency.
4) Time indices with the lowest densities (time occurrence rate, TOR) correspond to the “thinnest” portions of contour, and mark the time instants of zero-band frequencies.

The discussed schemes can be considered as projecting matrix $\Gamma$ elements onto frequency axis and time axis. The derived FOR and TOR reveal the distributions in both frequency and time, and can be effectively leveraged for extracting frequency trajectory along time. Since zero bands are always near the integer fraction values of nominal frequency, in practice, only those values should be scrutinized and with higher resolutions.

B. Envelope Extraction

When there is a variation in waveform amplitude, the correlation coefficients $\gamma_{ij}$ will manifest such dynamics. Fig. 8 shows the PW analysis results of an amplitude-modulated signal. It can be seen that frequency zero bands remain stable, while the $\gamma_{ij}$ values at other frequencies are oscillating.

The extraction of amplitude feature is performed by pulling out the elements of matrix $\Gamma$ that associates with a single-frequency component, revealing the trajectory of correlation intensity with respect to time. Practically, as long as the frequency is not zero-band frequency, oscillation patterns can be uncovered.

Note that $\gamma_{ij}$ values are in fact oscillating because of the periodical nature of correlation calculation (10). Hilbert transform [39] can be utilized in this situation to smooth out the oscillation and reveal the envelope of amplitude features.

C. Synchrophasor Estimation Paradigm

The prevalent synchrophasor algorithms inevitably utilize certain linearization in either time domain or frequency domain, so that the quantities of interest can be solved from linear matrix calculation, for instance, time-domain curve-fitting-based methods [11], [13]–[15], interpolated DFT [12]. In this paper, however, nonlinear fitting methods were utilized because of its high accuracy.

A generic model for PMU test signals is shown in (12)

$$
\chi(t) = \sqrt{2} X_{\text{rms}} [1 + k_{AM} \cos(2\pi f_{AM}t + \phi_{AM})] \\
\cdot \cos[2\pi f_c t + \pi R f t^2 + k_{FM} \cos(2\pi f_{FM} t + \phi_{FM}) + \phi_0]
$$

(12)
where $X_{\text{rms}}$ is the rms value of fundamental component, $k_{\text{AM}}/k_{\text{FM}}$ is the AM/FM level, $f_{\text{AM}}/f_{\text{FM}}$ is AM/FM frequency, $\phi_{\text{AM}}/\phi_{\text{FM}}$ is AM/FM angle, $f_s$ is input signal frequency, $R_f$ is frequency ramp level, and $\phi_0$ is initial phase angle. Fitting all the parameters in model (12) is unnecessary when not all power phenomena are prominent simultaneously. Therefore, the model (12) is simplified according to classified waveform type to increase computation efficiency. The idea of applying a signal model switching mechanism was originally proposed in [8], and the input waveform classification results from Sections II and III can be utilized so that only the parameters of interest are kept in the model. Although various degrees of simplification on (12) may be applied, typically four simplified models are of interest, as first discussed in [8], then elaborated recently in [17].

$$x(t) = \sqrt{2}X_{\text{rms}}\cos(2\pi f_s t + \phi_0)$$  \hspace{1cm} (13a)  
$$x(t) = \sqrt{2}X_{\text{rms}}(2\pi f_s t + \pi R_f t^2 + \phi_0)$$  \hspace{1cm} (13b)  
$$x(t) = \sqrt{2}X_{\text{rms}}[1 + k_m \cos(2\pi f_m t)\cos(2\pi f_s t + \phi_0)]$$  \hspace{1cm} (13c)  
$$x(t) = \sqrt{2}X_{\text{rms}}\cos(2\pi f_s t + k_d \cos(2\pi f_m t + \phi_0)).$$  \hspace{1cm} (13d)  

Equations (13a)–(13d) represents steady-state, frequency ramp, AM, FM cases, respectively. The switching mechanism can be conveniently realized by either “Case” structure in NI LabVIEW, or “Switch” structure in MATLAB.

Phasors are calculated using the Levenberg–Marquardt [40] algorithm (LMA), which are well-developed modules in both National Instruments (NI) LabVIEW and MATLAB. The inputs of LMA are: user-defined waveform models (13a)–(13d), initial values for unknown parameters, and raw sampled waveforms.

In order to boost computation efficiency and accuracy, prior information about the power grids should be leveraged. Due to large inertia of power grids, the value of frequency always fluctuates within a small range around nominal frequency, and also, consecutive frequency estimation results should be relatively close to each other. Amplitude initial value for phasor estimation (13) can be approximated using Hilbert transform [40] on raw data. Moreover, frequency initial value for phasor estimation (13) should be selected based on nominal values or previous estimation results.

V. HARDWARE PLATFORMS, TESTS, AND COMPARISON

A. Hardware Platforms and Test Plans

Two physical hardware platforms are considered for the proposed method: Schweitzer Engineering Laboratories (SEL) substation computer SEL-3355 and NI CompactRIO (cRIO)-9082. The platforms are considered since they are the typical equipment available for such uses in the field and research laboratories. A summary of the two platforms is given in Table I.

Although cRIO-9082 chassis is equipped with field-programmable gate array, the proposed method is implemented on the “host computer” in cRIO-9082 chassis which is running windows’ runtime system. To save computation time, data parallelism is employed in LabVIEW.
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Fig. 9. PW analysis on 55-Hz playback waveform. Highlighted frequency bands are identified using FOR and are within the vicinities of (a) 11 and 13.75 Hz, (b) 18.33 Hz, and (c) 27.5 Hz.

Fig. 10. PW analysis on frequency modulation playback waveform. Frequency extrema are identified by TOR, and frequency range is indicated by FOR.

Fig. 11. PW analysis on frequency ramp \( (R_f = 1 \text{ Hz/s}) \) playback waveform. Frequency extrema are identified by TOR, and frequency range is shown by FOR.

the frequencies spread out within a range, which is typical for dynamic waveforms with frequency variations. Frequency dynamic patterns can clearly be observed from the extracted frequency features.

2) Simulation Analysis of Envelope Extraction: Simulations on waveforms with various amplitude modulation frequencies are conducted. \( \Gamma \) matrix elements associated with 55- and 60-Hz PW frequencies are extracted, and the envelopes are calculated using Hilbert transform. The envelope trajectories are depicted in Fig. 12. It can be observed that the envelopes are oscillating, implying amplitude oscillations.

3) Synchrophasor Estimation Paradigm Demonstration: ith identified input waveform type, the paradigm in Section IV-C can be applied. The results are shown in Table III. It can be observed that all the results are within 1/4 of the IEEE standard [5] requirements for both P- and M-class PMU.

To summarize, identifying waveform type in advance can notably improve synchrophasor estimation accuracy. Detailed discussions on this paradigm can be found in [8] and [17]. In practice, however, as implied in Fig. 1, the selection of synchrophasor algorithm is up to the developers.

C. Simulink Simscape Power Systems Simulation Waveforms

In this section, Simulink Simscape Power Systems simulation data are generated as test waveforms, as they are considered close to what can be actually observed in the power grid. Disturbances are introduced to a steady-state power grid to induce dynamics in Simulink Simscape Power Systems package. Inarguably, the simulated waveforms, as well as real power grid waveforms, are always a combination of modulations, frequency drift, harmonics and noise, even though some of the patterns may appear to be predominant.

1) Amplitude Modulated Waveform: The classic Kundur’s interarea system [41] is used as the test case. A phase C-ground fault in the middle of Line 1 is added and is cleared in 5 cycles, triggering an interarea oscillation. The oscillation is primarily an amplitude modulation. Fig. 13 shows the phase A voltage on Line 2 near Area2.

The spectra of input waveform in two windows are analyzed, shown in Fig. 14. It can be clearly seen that after 12.5 s, a new mode is induced at around 56 Hz.

2) Amplitude and Frequency Modulation Waveform: The 29-bus system [42] is used as the test system. A phase B–C
fault in North-West Network at the primary winding of the 2200-MVA transformer (near LG3) is added and cleared in 10 cycles. The terminal voltages and frequency of generator B_7 MAN 5000 MVA are observed, as shown in Fig. 15.

Both amplitude and frequency fluctuations can be observed, and the spectrum of waveform between 4.5 and 12.5 s is analyzed and shown in Fig. 16. It can be observed that the system is operating at off-nominal frequency while experiencing both amplitude and frequency modulation.

3) Frequency Ramp Waveform: The 29-bus system [42] is used as the test system. The MTL load (15 500 MW) on Bus MTL7 is tripped off the system at 1 s, causing the system frequency to ramp up. The terminal voltages and frequency of generator B_7 MAN 5000 MVA are observed, as shown in Fig. 17.

The waveform from 1 to 5 s is studied in the spectrogram (STFT), shown in Fig. 18, and a roughly linear frequency chirp can clearly be seen in the spectrogram.
D. Algorithm Evaluation and Comparison Using Simulink-Simulated Data

The waveforms in Section V-B are used as test waveforms. The emphasis of PW and waveform classification algorithm is put on their performances with respect to dynamic waveforms. Both the efficacy and efficiency are tested, illustrated, and/or tabulated. Assuming the power system operates at around nominal frequency before disturbances, only the zero band around 30 Hz is analyzed at a resolution of 0.01 Hz.

Comparison is made with the WT-based feature extraction method, where the “time–frequency analysis” block in Fig. 8 is replaced with WT. Simulations reveal that the family of mother wavelet does not dramatically change the performance of WT-based methods, and thus Daubechies-2 (db2) wavelet [43] is selected to demonstrate because of its simplicity.

Due to the large inertia of the power grid, a longer analyzing window will inarguably yield more clear and convincing classification results. The use of a slightly longer window is justified in the discussion of the overall hierarchy in Fig. 1. In this section, a window length of 12 cycles is used for envelope extraction for amplitude modulation waveform, while 10-cycle windows are used for frequency trajectory extraction. In order to improve efficiency while maintaining adequate accuracy, the data in the data buffer are downsampled before analysis. $\gamma_{ij} = (20 \pm 8) \times 10^{-5}$ is used as the threshold values to extract FOR and TOR.

The proposed algorithms are tested and timed in both MATLAB (SEL-3355) and NI LabVIEW (NI cRIO) environments. The WT-based method is implemented in NI LabVIEW (NI cRIO) to serve as a comparison. Data parallelism is employed in NI LabVIEW programming.

1) Envelope Extraction Under Amplitude Modulation: The data from buffer are further downsampled to 500 Hz for envelope extraction. Fig. 19 shows the envelope extraction comparison between the PW method and the WT method. The 55-Hz component is extracted, and 12 cycles are used as the analyzing window in order to effectively reveal the fluctuation in amplitude.

As can be seen, both PW and WT methods are able to exhibit amplitude modulation dynamics. This is because the amplitude dynamics manifest variations on correlation coefficients calculated from either PW- or WT-based methods. The computation times are listed in Table IV.

2) Frequency Trajectory Extraction Under Combined Amplitude and Frequency Modulation: In this test, frequency trajectory extraction is evaluated. The data in the buffer are downsampled to 1 kHz. The results are shown in Fig. 20. It can be clearly observed from Fig. 20(b) that the frequency is undergoing modulation. Using the FOR and TOR methods discussed in Section V, the frequency features can be extracted and are shown in Fig. 20(a) as the blue dots. The blue dots are, in fact, the outline of the dark area in Fig. 20(b). Through smoothing and interpolation, the frequency trajectory can be extracted, and shown as the orange curve in Fig. 20(a).

The time–frequency analysis using db2 mother wavelet is also performed, and the result is shown in Fig. 21. Computation time aside, the extracted frequency contours do not

<table>
<thead>
<tr>
<th>Platform</th>
<th>MATLAB (SEL-3355)</th>
<th>LabVIEW (NI cRIO-9082)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedure</td>
<td>PW Method</td>
<td>PW Method</td>
</tr>
<tr>
<td>Sampling Rate</td>
<td>500Hz</td>
<td>500Hz</td>
</tr>
<tr>
<td>Data Length</td>
<td>12 cycles</td>
<td>12 cycles</td>
</tr>
<tr>
<td>Time-Frequency Analysis</td>
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<td>2ms</td>
</tr>
<tr>
<td>Envelope Extraction</td>
<td>8ms</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>16ms</td>
<td>2ms</td>
</tr>
</tbody>
</table>

* Includes the formation of correlation intensity matrix $T$ using PW or WT method
** Includes envelope extraction using Hilbert Transform method
clearly show any sign of frequency fluctuations. A separate band, however, on the higher scale can be observed, due to the higher frequency components from the modulation. Overall, WT results cannot be used to extract more frequency dynamic behavior. The computation times are listed in Table V.

3) Frequency Trajectory Extraction Under Frequency Ramp: In this test, frequency trajectory extractions are evaluated. The data from buffer are further downsampled to 1 kHz. The results are shown in Fig. 22. From Fig. 22(b), it is obvious that the frequency is ramping up. Similarly, the outline of PW results is extracted using the FOR and TOR methods, and is shown as the blue dots in Fig. 22(a). The frequency trajectory can be extracted then by utilizing smoothing and interpolation, and is shown as the orange curve, which is nearly linear.

The time–frequency analysis using db2 mother wavelet is also conducted, and shown in Fig. 23. Similarly, there is no pattern showing a clear frequency variation, and thus cannot be used further to study frequency trajectory. Also, a separate band at higher scale can be observed, due to the higher frequency components in frequency ramp waveform. Similar to the previous scenario, WT results do not depict enough information to identify frequency dynamics. The computation times are shown in Table VI.

VI. CONCLUSION

A novel multiresolution time–frequency analysis method was designed for power waveform classification, and is further leveraged to implement accurate reference synchrophasor estimation. The conclusions are as follows.

1) A new framework for synchrophasor estimation is proposed, where the waveform type is identified first. With the proposed framework, both the efficiency and accuracy of synchrophasor estimation will be improved.

2) A novel time–frequency multiresolution analysis method based on PWs is proposed. The new PW method is capable of tracking the variations of frequency components in the waveform.

3) A novel waveform classification method utilizing the results of PW multiresolution analysis is introduced. The waveform envelope and frequency trajectory are extracted and leveraged to classify the type of input waveforms.

4) The proposed PW method and waveform classification method are implemented in practical hardware platforms, i.e., SEL substation computer, and NI CompactRIO.
5) Extensive tests are conducted to evaluate the efficacy and efficiency of proposed methods in both platforms. The test results show that the proposed methods are capable of efficiently extracting the amplitude and frequency feature of input waveform and perform classification, even with the presence of noise and harmonics.

REFERENCES


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