Abstract—DC offset has significant effect on extracting fundamental frequency components. It directly impacts the accuracy of the fundamental frequency component based protective relaying algorithms. This paper proposes a novel method for estimating the fundamental phasor while eliminating the dc offset using improved recursive wavelet transform. The proposed approach converges to the actual value in less than one cycle, and the computation burden is fairly low because of the recursive formula. Studies indicate that to achieve a certain level of accuracy, the higher sampling frequency one uses, the shorter data window the computation needs, and vice versa. Comparing with conventional DFT filter, simulation results demonstrate that the proposed phasor estimation method achieves better performance.

Index Terms—protective relaying, fundamental frequency component, phasor estimation, dc offset, improved recursive wavelet transform, data window, conventional DFT filter

I. INTRODUCTION

In protective relaying application, Discrete Fourier Transform (DFT) is widely used as a filtering algorithm for extracting fundamental phasors [1], [2]. Conventional DFT algorithm achieves excellent performance when the signals contain only fundamental frequency and integer harmonic frequency components. Since in most cases the currents contain DC offsets this may introduce fairly large errors in the estimation of fundamental frequency phasor [3], [4].

Many techniques have been proposed to eliminate the DC offset in waveforms. A digital mimic filter based method was proposed in [5]. This filter features high-pass frequency response which results in bringing high frequency noise to the outcome. It performs well when its time constant matches the time constant of the exponentially decaying component. Theoretically, the decaying component can be completely removed from the original waveform once its parameters can be obtained. Based on this idea, [6], [7] utilize additional samples to calculate the parameters of the decaying component. Reference [8] uses the simultaneous equations derived from the harmonics. A new Fourier algorithm and three simplified algorithms based on Taylor expansion were proposed to eliminate the decaying component in [9]. In [10], author estimates the parameters of the decaying component by using the phase angle difference between voltage and current.

This method requires both voltage and current inputs. As a result, it is not applicable to the current-based protection schemes.

Recursive wavelet approach has been introduced in protective relaying for a long time [11]-[13]. The improved model with single-direction recursive equations is more suitable for the application to real-time signal processing [14]. The band energy of any center frequency can be extracted through improved recursive wavelet transform (IRWT) with moderately low computation burden. Recursive wavelet features band-pass filter achieves good performance in suppressing the sub-harmonic components. A novel filtering approach using IRWT is proposed for estimating the fundamental frequency phasor while eliminating the effect of dc offset.

This paper first introduces the recursive wavelet transform and its characteristics in the time and frequency domain. The phasor estimation and DC offset removal algorithm are presented next. The relationship between the convergence and the sampling rate is studied as well. It indicates that to achieve a certain level of accuracy, the higher sampling rate one uses, the shorter data window it needs, and vice versa. Simulation results demonstrate the effectiveness of the proposed phasor estimation method. It converges to the actual value in less than one cycle, and the computation burden is fairly low because of the recursive formula, thus it can satisfy the time response requirement of the high speed relaying schemes.

II. FILTERING ALGORITHM

A. Characteristics of Conventional DFT Filters

In terms of the length of the data window used for the filtering calculation, conventional Discrete Fourier Transform (DFT) can be classified into two categories: Full Cycle DFT (FCDFT) and Half Cycle DFT (HCDFT). Frequency responses of FCDFT and HCDFT shown in Fig. 1 and Fig. 2 respectively indicate the performance in suppressing integer frequency harmonics. The sub-harmonic components and DC offset can not be easily eliminated with conventional DFT filters. This can be seen from Fig. 3, which presents the frequency spectrum of a set of exponentially decaying signals with a broad range of time constants (0.5 to 5 cycles). In power system, DC offset widely exists in voltage and current signals when various disturbances occur, such as fault or oscillation, and it may cause the calculated amplitude deviate from the real value 15% in worst case [9].

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B. Improved Recursive Wavelet Transform

The mother wavelet is given as:

$$\psi(t) = \left( -\frac{\delta^2}{3} - \frac{\delta^4}{6} - \frac{\delta^5}{15} \right) e^{i\delta x + \omega_0 t} u(-t)$$

A set of wavelet functions can be created by dilating and shifting the mother wavelet, as given below:

$$\psi_{a,b}(t) = a^{-1/2} \cdot \psi\left(\frac{t-b}{a}\right)$$

The wavelet coefficient for a given signal $x(t)$ can be expressed as below:

$$W_{x(t)}(a,b) = a^{-1/2} \int_{-\infty}^{\infty} x(t) \cdot \psi\left(\frac{t-b}{a}\right)^* dt$$  \hspace{1cm} (1)

Assign $\delta = 2\pi / \sqrt{3}$, $\omega_0 = 2\pi$ thus $\psi(t)$ is admissible. Besides, we have $a = 1 / f$, that is the scale coefficient $a$ is reciprocal to the frequency $f$ used in the analysis.

Improved Recursive Wavelet (IRW) exhibits good time-frequency characteristics. Fig. 4 shows its real part and imaginary part in time domain. The real part, imaginary part and magnitude of the frequency characteristics are given in Fig. 5, where $a = 1 / f_0$. As we can see in Fig. 5, IRW features a band-pass filter with the center frequency $f_0$.

Fig. 4. Waveforms of IRW in time domain

$$W_{x(k)}^{IR}(f,k) = \Delta T \sqrt{f} \{\lambda_1 x(k-1) + \lambda_2 x(k-2) + \lambda_3 x(k-3) + \lambda_4 x(k-4) + \lambda_5 x(k-5)\}$$

$$- \beta W_{x(k)}^{IR}(f,k-1) - \beta W_{x(k)}^{IR}(f,k-2) - \beta W_{x(k)}^{IR}(f,k-3) - \beta W_{x(k)}^{IR}(f,k-4) - \beta W_{x(k)}^{IR}(f,k-5) - \beta W_{x(k)}^{IR}(f,k-6)$$

where,

$$\sigma = e^{-\beta \Delta T (a-\omega_0)}$$

$$\lambda_1 = \sigma[(a\Delta T)^3 / 3 - (a\Delta T)^4 / 6 + (a\Delta T)^5 / 15]$$

$$\lambda_2 = \sigma^2[2(a\Delta T)^3 / 3 - 5(a\Delta T)^4 / 3 + 26(a\Delta T)^5 / 15]$$

$$\lambda_3 = \sigma^3[-6(a\Delta T)^3 / 3 + 22(a\Delta T)^4 / 5]$$

$$\lambda_4 = \sigma^4[2(a\Delta T)^3 / 3 + 5(a\Delta T)^4 / 3 + 26(a\Delta T)^5 / 15]$$

$$\lambda_5 = \sigma^5[(a\Delta T)^3 / 3 + (a\Delta T)^4 / 6 + (a\Delta T)^5 / 15]$$

$$\beta_1 = -6\sigma, \beta_2 = 15\sigma^2, \beta_3 = -20\sigma^3$$

$$\beta_4 = 15\sigma^4, \beta_5 = -6\sigma^5, \beta_6 = \sigma^6$$

Assume the input signal $x(k)$ and the sampling period $\Delta T$, the formula of improved recursive wavelet transform is given as follows:
III. PHASOR ESTIMATION SCHEME

A. Estimating Phasor Using IRWT

Consider a sinusoidal signal expressed in complex form:

\[ x(t) = A_m e^{j(\omega t + \theta)} \quad t \geq 0 \]

where \( A_m \) is the amplitude, \( \theta \) is the phase angle.

Apply IRWT to signal \( x(t) \) using (1). As derived in Appendix, we obtain,

\[ W_{x(t)}(a, b) = A_m e^{j(ab + \theta)} \cdot I(a, b) \tag{3} \]

Assume the sampling frequency \( f \) equals \( N \) times the fundamental frequency \( f_0 \). That is \( f = N f_0 \) and the sample period is \( \Delta T = 1/f \). The discrete expression of equation (3) is:

\[ W_{x(k)}(a, k) = A_m e^{j(a k \Delta T + \theta)} \cdot I(a, k) \]

To extract the fundamental component, simply let \( a = 1/f_0 \). From (2) we have the coefficient \( W^{IR}_{x(k)}(a, k) \). Select proper sampling rate so that the error resulting from the discrete data computation is within the limit of tolerance, we have \( W_{x(k)}(a, k) = W^{IR}_{x(k)}(a, k) \). That is:

\[ A_m e^{j(2\pi k/N + \theta)} = W^{IR}_{x(k)}(a, k) / I(a, k) \]

Thus,

\[ A_m = \left| W^{IR}_{y(k)}(a, k) / I(a, k) \right|, \quad \theta = \phi - 2\pi k / N \tag{4} \]

In above formula, \( W^{IR}_{x(k)}(a, k) \) is the wavelet transform coefficient calculated by the recursive equation (2). \( I(a, k) \) is constant given in Appendix which can be calculated in advance.

B. Eliminating Exponentially Decaying Component

Consider a signal consisting of exponentially decaying component,

\[ y(t) = D_r e^{-\tau/t} + x(t) \quad t \geq 0 \]

where \( D_r \) is the amplitude, \( \tau \) is the time constant.

Apply IRWT to signal \( y(t) \) using (1). The IRWT coefficient of signal \( y(t) \) derived in Appendix is given as

\[ W_{y(t)}(a, b) = I_r(a, b) \cdot D_r e^{-b/\tau} + W_{x(t)}(a, b) \]

As we can see from above equation, \( D_r \) and \( \tau \) can be calculated by subtracting \( I_r(a, b) \cdot x(t) \) from \( W_{x(t)}(a, b) \). Introducing the wavelet transform coefficient of signal \( y(t) \) by applying formula (2) with \( a = 1/f_0 \), in discrete form, we have

\[ \begin{align*}
W^{IR}_{y(k)}(a, k) - I(a, k) \cdot y(k) & = [I_r(a, k) - I(a, k)] \cdot D_r e^{-\tau/k} \\
\end{align*} \]

Designate

\[ g(\tau, k) = I_r(a, k) - I(a, k) \]

\[ h(k) = W^{IR}_{y(k)}(a, k) - I(a, k) \cdot y(k) \]

We obtain

\[ g(\tau) \cdot D_r e^{-\tau/k} = h(k) \tag{5} \]

Introduce additional sample \( y(k+1) \)

\[ g(\tau) \cdot D_r e^{-\tau/(k+1)} = h(k+1) \tag{6} \]

where

\[ h(k+1) = W^{IR}_{y(k+1)}(a, k+1) - I(a, k) \cdot y(k+1) \]

From (5) and (6), we obtain,

\[ \tau = \frac{\Delta T \log h(k) / h(k+1)}{D_r \cdot g(\tau) \cdot e^{-\tau/k}} \]

It should be noticed that \( g(\tau) \) is known after \( \tau \) is calculated. Then, the decaying component can be removed completely by the following formulae:

\[ A_m e^{j(2\pi a k/N + \theta)} = \left| W^{IR}_{y(k)}(a, k) - I_r(a, k) \cdot D_r e^{-\tau/k} \right| / I(a, k) \tag{7} \]

C. Analysis of the Impact of Data Window Length

Theoretically, the amplitude and phase angle of the input signal can be accurately estimated using (7) by two samples, which means the length of the data window would be \( 2 \cdot \Delta T \). In reality, two factors should be considered when selecting the data window length. One is that formula (4) and (7) are derived based on the assumption that the error resulting from the discrete computation is negligible. Another is the error introduced by the recursive calculation.

In practical application, the length of the data window should be selected in terms of the sampling frequency. Fig. 6 presents the convergence characteristics of the proposed filtering algorithm vs. the sampling frequency. In Fig. 6, the window length is one cycle of the fundamental frequency; \( f \) is the sampling frequency while \( f_0 \) is the fundamental frequency.

![Fig. 6. Convergence characteristics vs. sampling frequency](image-url)
shortens the data window and expedites the convergence process for the phasor estimation.

Fig. 7. Sampling frequency vs. data window length

IV. PERFORMANCE TEST

In this section, an input signal comprising fundamental frequency component and exponentially decaying component is used to test the performance of the proposed phasor estimate and DC offset elimination schemes. The time constant $\tau$ is studied over a broad range 0.5–5 cycles. The results are compared with the conventional DFT (full cycle and half cycle) method. Select $N = 400$, $T_s = 0.75$ cycle, $f_0 = 60$ Hz, $A_m = 1$ p.u., $\theta = 60^\circ$. Consider the severe condition, that is the fundamental frequency component and decaying component have the same amplitude. The input signal is described in complex form:

$$y(k) = A_m e^{-k \Delta t / \tau} + A_m e^{j(\pi k / 200 + \theta)}$$

Apply formula (7) and obtain the amplitude and phase angle of the input signal $y(k)$. Fig. 8 and Fig. 9 give the calculated amplitudes and phase angles in four cycles over $\tau = 1.0$ cycle respectively using the full cycle DFT, half cycle DFT and the proposed algorithm. The convergence time of the proposed algorithm is 0.75 cycle, and the calculation errors are 0.2662% and 0.2138% for amplitude and phase angle respectively.

Fig. 8 Amplitude of the estimated phasor

Fig. 9 Phase angle of the estimated phasor

Table I summarizes the test results, in which the estimated values are obtained at half cycle, full cycle and 0.75 cycle for HCDFT, FCDFT and proposed filter respectively. The phase error is calculated in full scale ($360^\circ$).

<table>
<thead>
<tr>
<th>$\tau$ (cycle)</th>
<th>Filter Type</th>
<th>Estimate Value</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_m$ (p.u.)</td>
<td>$\theta$ (deg)</td>
<td>$\text{Err}_{A_m}$</td>
</tr>
<tr>
<td>0.5</td>
<td>HCDFT</td>
<td>0.7623</td>
<td>5.6721</td>
</tr>
<tr>
<td></td>
<td>FCDFT</td>
<td>0.8473</td>
<td>46.6454</td>
</tr>
<tr>
<td></td>
<td>Proposed</td>
<td>1.0034</td>
<td>59.2123</td>
</tr>
<tr>
<td>1</td>
<td>HCDFT</td>
<td>0.6796</td>
<td>-11.1511</td>
</tr>
<tr>
<td></td>
<td>FCDFT</td>
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<td>51.4980</td>
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<td>1.0027</td>
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<tr>
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<td>FCDFT</td>
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<tr>
<td></td>
<td>Proposed</td>
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<tr>
<td>3</td>
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</tr>
<tr>
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<td></td>
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<td>FCDFT</td>
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<td>1.0024</td>
<td>59.1788</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

DC offset has significant effect on extracting the fundamental frequency components. Conventional DFT filters cannot easily eliminate it when estimating the components of interest. This directly impacts the accuracy of the fundamental frequency component based protective relay algorithms. This paper proposes a novel method for estimating the fundamental phasor and eliminating the DC offset using improved recursive wavelet transform. Studies indicate that the convergence of proposed algorithm is related to the sampling frequency. To achieve a certain level of accuracy, the higher sampling rate one uses, the shorter data window it needs, and vice versa. The proposed method converges to correct results within one cycle, and the computation burden is fairly low because of the recursive formula. Comparing with conventional DFT filter, performance tests demonstrate that the proposed method achieves very good results.
The IRWT coefficient of the input signal $x(t)$ is:

$$W_{y(t)}(a,b) = a^{-1/2} \int_{-\infty}^{\infty} x(t) \cdot \psi^* \left( \frac{t-b}{a} \right) dt \quad b \geq 0$$

$$= a^{-1/2} \int_{0}^{1} A_m e^{-j(\omega t + \theta)} \cdot [\left( -\frac{\delta^3}{3} \left( t - \frac{b}{a} \right)^3 - \frac{\delta^4}{6} \left( t - \frac{b}{a} \right)^4 \right) - \frac{\delta^5}{15} \frac{b-a}{a} \left( \delta - j\omega \right) (t-b) a \cdot \eta_1 dt$$

Denote

$$t = k \cdot a + b, \ k \in [-b/a, 0]$$

$$\eta_1 = \delta + f(a \omega - \omega_0)$$

We have

$$W_{y(t)}(a,b) = A_m e^{-j(\omega t + \theta)} \cdot I(a,b)$$

where

$$I(a,b) = \sqrt{a} \cdot \left( -\frac{\delta^3}{3} \frac{\eta_2}{\eta_1} \cdot e^{\frac{\eta_2}{\eta_1} a} + \frac{3\delta^2}{\eta_1} \right)$$

$$\cdot e \left( \frac{\delta^3}{\eta_1} - \frac{6\delta^4}{a \eta_1} \cdot e^{\frac{\delta^4}{\eta_1} a} - \frac{6\delta^5}{a^2 \eta_1} \cdot (1 - e^{\frac{\delta^5}{\eta_1} a}) \right)$$

$$- \frac{\delta^4}{6} \left( \frac{1}{\eta_1} - \frac{\delta^3}{\eta_1} \frac{e^{\frac{\delta^3}{\eta_1} a}}{\eta_1} + \frac{\delta^4}{a \eta_1} \cdot e^{\frac{\delta^4}{\eta_1} a} \right)$$

$$- \frac{12\delta^5}{\eta_1} \cdot e^{\frac{\delta^5}{\eta_1} a} + \frac{24\delta^6}{\eta_1} \cdot e^{\frac{\delta^6}{\eta_1} a}$$

$$\cdot \left( 1 - e^{\frac{\delta^6}{\eta_1} a} \right) + \frac{24}{\eta_1} \cdot e^{\frac{\delta^6}{\eta_1} a} + \frac{5\delta^7}{\eta_1} \cdot e^{\frac{\delta^7}{\eta_1} a} - \frac{20\delta^8}{\eta_1^3} \cdot e^{\frac{\delta^8}{\eta_1^3} a}$$

$$+ \frac{60\delta^9}{\eta_1^3} \cdot e^{\frac{\delta^9}{\eta_1^3} a} - \frac{120\delta^{10}}{\eta_1^3} \cdot e^{\frac{\delta^{10}}{\eta_1^3} a} - \frac{120\delta^{11}}{\eta_1^3} \cdot (1 - e^{\frac{\delta^{11}}{\eta_1^3} a}) \right)$$

Similarly as deriving the wavelet coefficient of signal $x(t)$, for signal $y(t)$, denote $\eta_2 = \delta - \frac{\tau}{\tau} j \omega_0$, we have

$$W_{y(t)}(a,b) = a^{-1/2} \int_{0}^{1} y(t) \cdot \psi^* \left( \frac{t-b}{a} \right) dt$$

where $I(a,b)$ has the same with $I(a,b)$ by replacing $\eta_1$ with $\eta_2$.