# A Comprehensive Contribution Factor Method for Congestion Management

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*Abstract*-- **This paper introduces a comprehensive method for congestion management by using network, generator and load contribution information. When congestion occurs, the priority is given first to the network contribution factor, then to the generator contribution factor and last to the load contribution factor. If the congestion can be relieved by the network adjustment, only the network control is used. Otherwise, generator re-dispatching is initialized. The method may also be combined with demand side (load) management to solve the congestion. The congestion management scheme is presented in this paper.**

***Index Terms*—Network Contribution Factor, Generator Contribution Factor, Load Contribution Factor, Congestion Management, Overload Relieving, FACTS**

## I. Introduction

One of the most challenging problems for a competitive power environment is that congestion may occur frequently [1], which hinders the transmission open access and finally impairs functioning of the electricity market. There are many papers in the literature talking about congestion management [2,3,4,5]. However, many of them focus on the cost and responsibility allocation. There are fewer papers researching the problem from the engineering point of view [6].

This paper presents a congestion management scheme by using network, generator and load contribution factors to solve the congestion problem. When congestion occurs at one or several lines, the Flow Network Contribution Factor (FNCF) is calculated and ranked for the purpose of network control to solve the congestion. If network control is not enough to relieve the congestion, both Flow Generator Contribution Factor (FGCF) and Flow Load Contribution Factor (FLCF) are calculated. Either generator re-dispatching alone or combined with demand side management due to the availability of load management is chosen to solve the congestion problem. A congestion-solving scheme is presented and its result is checked with the power flow result.

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This paper introduces the mathematical formulation of the network, generator and load contribution factor method in Section II. The congestion-solving scheme is described in Section III. Numerical test results are presented in Section IV. Conclusion and references are given in Section V and VI respectively.

## II. MATHEMATICAL FORMULATION

### A. Flow Network Contribution Factor

From the fast decoupled power flow, we know the approximate real power equation based on the simple fact that the line resistance is much smaller than the reactance, $r_i << x_i$

$$\frac{P}{E} = B^{'}\theta \quad (1)$$

where, P, E, $\theta$ are the node real power injection, magnitude and angle of the bus voltage respectively.

$$(B^{'})_{ij} = -b_{ij} \quad (2)$$

where $b_{ij}$ is the series inductance of the line i-j

Given an n-bus-l-branch system, A is the node-branch incidence matrix, $Y_p$ is the primitive branch admittance matrix, $Y_{bs}$ is the node shunt capacitance matrix,

$$Y_p = diag[y_1 \quad ... \quad y_l] \quad (3)$$

$$Y_{bs} = diag[y_{s1} \quad ... \quad y_{sn}] \quad (4)$$

$$A_{ij} = \begin{cases} 1 & \text{i is the sending node of branch j} \\ -1 & \text{i is the receiving node of branch j} \\ 0 & else \end{cases} \quad (5)$$

Bus admittance matrix can be obtained from:

$$Y = AY_pA^T + Y_{bs} \quad (6)$$

Assign $B^{'}$ as the negative value of the imaginary part of $Y$ matrix,

$$B^{'} = -imag(Y) \quad (7)$$

For the real power flow,

$$P_{line} = -imag(Y_p)A^T(E\theta) \quad (8)$$

The approximate node injection can be composed from the line flows associated with this node,

$$P_{node} \cong AP_{line} = A(-imag(Y_p))A^T(E\theta) \quad (9)$$

For the parameter variance of line i, $\Delta y_i$, assume that the node injection $P_{node}$ and bus voltage magnitude E do not change much, and the bus voltage angle $\theta$ varies $\Delta\theta$. Then assign

$$Y_1 = -imag(Y_p) \quad (10)$$

Since the node injection does not change, from (9) we get,

$$A(Y_1 + \Delta Y_1)A^T E(\theta + \Delta\theta) = AY_1 A^T E\theta \quad (11)$$

thus,

$$\Delta\theta = -(A(Y_1 + \Delta Y_1)A^T)^{-1}(A\Delta Y_1 A^T)\theta \quad (12)$$

where $\Delta Y_1 = diag[0 \ \ .. \ \ \Delta y_i \ \ .. \ \ 0]$ (13)

From (8), we get the line flow change,

$$\begin{aligned}\Delta P_{line} = P_{line}^{new} - P_{line} = \Delta Y_1 A^T (E\theta) - \\ (Y_1 + \Delta Y_1)A^T (A(Y_1 + \Delta Y_1)A^T)^{-1}(A\Delta Y_1 A^T)(E\theta)\end{aligned} \quad (14)$$

since

$$\Delta Y_1 A^T (E\theta) = \begin{bmatrix} 0 & .. & \sum_{j=1}^{n} A_{ji} E_j \theta_j & .. & 0 \end{bmatrix}^T \Delta y_i \quad (15)$$

$$(A\Delta Y_1 A^T)(E\theta) = Ki\Delta y_i \quad (16)$$

where

$$Ki = \begin{bmatrix} A_{1i}\sum_{j=1}^{n} A_{ji}E_j\theta_j & .. & A_{ii}\sum_{j=1}^{n} A_{ji}E_j\theta_j & .. & A_{ni}\sum_{j=1}^{n} A_{ji}E_j\theta_j \end{bmatrix}^T \quad (17)$$

$$X_1 = (A(Y_1 + \Delta Y_1)A^T)^{-1} \quad (18)$$

$$X = (AY_1 A^T)^{-1} \quad (19)$$

we can get the following line flow variance equations,

for line k, $k \neq i$,

$$\Delta P_{line-k} = -[A_{1k} \ \ \ldots \ \ A_{nk}]X_1 Ki(y_k \Delta y_i) \quad (20)$$

for line k, $k = i$,

$$\Delta P_{line-i} = \left(\sum_{j=1}^{n} A_{ji}E_j\theta_j / y_i' - [A_{1i} \ \ \ldots \ \ A_{ni}]X_1 Ki\right)(y_i' \Delta y_i) \quad (21)$$

where

$$y_i' = y_i + \Delta y_i \quad (22)$$

The Flow Network Contribution Factor (FNCF) $N_f$ can be defined as follows,

for line k, $k \neq i$

$$N_{f,k} = -[A_{1k} \ \ \ldots \ \ A_{nk}]X_1 Ki \quad (23)$$

for line k, $k = i$

$$N_{f,k} = \sum_{j=1}^{n} A_{ji}E_j\theta_j / y_i' - [A_{1i} \ \ \ldots \ \ A_{ni}]X_1 Ki \quad (24)$$

In general, bus impedance matrix (imaginary part) $X_1$ doesn't change much from the original matrix $X$ due to the line parameter variance $\Delta y_i$. If we assume $X_1 \cong X$, the Flow Network Contribution Factor $N_f$ will be constant except for line i.

We can easily get the line flow variance,

$$\Delta P_{line-k} = N_{f,k} y_k \Delta y_i, \ \text{k=1,…,l} \quad (25)$$

From (25) we can see that the line flow variances are related to three components: the Network Contribution Factor $N_f$, this line's series inductance, and admittance variance of the line i. For each line parameter change, we can get all other line flow variances easily. Vice versa, we can calculate the line parameter variance based on the exact line flow change we want. This gives us a good guidance to issue network control to re-dispatch the line flows:

**Step 1**, use the base network $X$ matrix to get $N_f$ and $\Delta P_{line-k}$, so the positive or negative contribution of each line parameter change to the line flow of interest is easily known.

**Step 2**, choose the most contributable line, change its parameter, get the actual $N_f$ and $\Delta P_{line-k}$ by using the real $X_1$ and $y_i'$, thus get the desired overload relieving.

**Step 3**, run power flow program to verify the result.

For the line switching, just simply assign $\Delta y_i = -y_i$, $\Delta P_{line-i} = -P_{line-i}$. For other line flow changes, for a small size system, switching off an in-service line may make a big variance of $X$, so we use real $X_1$ and $y_i'$ to get the actual $N_f$ and $\Delta P_{line-k}$. For a large size system, switching off one or several lines may not change $X$ as much, so we still use the three steps above.

The above method can give quick guidance for selection of the parameter to change and the exact parameter variance. The power flow calculation can verify it to get an accurate control.

### B. *Generator Contribution Factor* [6]

Let the gross nodal power $P_i^g$ flowing through node i (when looking at the inflows) be defined by

$$P_i^g = \sum_{j \in \alpha_i^u} \left|P_{ij}^g\right| + P_{Gi} \quad \text{for i =1,2,…,n} \quad (26)$$

where $\alpha_i^u$ is the set of nodes supplying power and directly connected into node i, and $P_{Gi}$ is the power generation injected into node i. Under normal situations, we can rewrite the equation as:

$$P_i^g - \sum_{j \in \alpha_i^u} \frac{\left|P_{ji}^g\right|}{P_j} P_j^g = P_{Gi} \quad \text{or} \quad A_u P_{gross} = P_G \quad (27)$$

where $A_u$ is the upstream distribution matrix with its (ij)th element defined by

$$\left[A_u\right]_{ij} = \begin{cases} 1 & for\ i = j \\ -\left|P_{ji}\right| / P_j & for\ j \in \alpha_i^u \\ 0 & other \end{cases} \quad (28)$$

where $P_{ji}$ is the real power flow from node j to node i in line j-i; $P_j$ is the total real power injected into node j. Then we have

$$P_i^g = \sum_{k=1}^{n} [A_u^{-1}]_{ik} P_{Gk} \quad \text{for i=1,2,…,n} \quad (29)$$

Finally the contribution of each generator to line i-j flow can be calculated by

$$P_{ij}^g = \sum_{k=1}^{n} D_{ij,k}^g P_{Gk} \quad \text{for} \ \ j \in \alpha_i^u \quad (30)$$

where $D_{ij,k}^g = P_{ij}^g [A_u^{-1}]_{ik} / P_i^g$ is called the topological generation distribution factor, and the contribution of generator k to line i-j is equal to $D_{ij,k}^g P_{Gk}$.

### C. *Load Contribution Factor* [6]

Similarly, let the gross nodal power $P_i^n$ (looking from outflows) be defined by

$$P_i^n = \sum_{j\in\alpha_i^d} \left|P_{ij}^n\right| + P_{Li} \quad \text{for i =1,2,…,n} \tag{31}$$

where $\alpha_i^d$ is the set of nodes supplied directly from node i, and $P_{Li}$ is the load at node i. Similarly, we can rewrite the equation as:

$$P_i^n - \sum_{j\in\alpha_i^{du}} \frac{\left|P_{ji}\right|}{P_j} P_j^n = P_{Li} \quad \text{or} \quad A_d P_{net} = P_L \tag{32}$$

where $A_d$ is the downstream distribution matrix with its (ij)th element defined by

$$\left[A_d\right]_{ij} = \begin{cases} 1 & for\ i = j \\ -\left|P_{ji}\right| / P_j & for\ j \in \alpha_i^d \\ 0 & other \end{cases} \tag{33}$$

$$P_i^n = \sum_{k=1}^{n} [A_d^{-1}]_{ik} P_{Lk} \quad \text{for i=1,2,…,n} \tag{34}$$

Finally the contribution of each load to line i-j flow can be calculated by

$$P_{ij}^n = \sum_{k=1}^{n} D_{ij,k}^n P_{Lk} \quad \text{for } j \in \alpha_i^d \tag{35}$$

where $D_{ij,k}^n = P_{ij}^n [A_d^{-1}]_{ik} / P_i^n$ is called the topological load distribution factor, and the contribution of load k to line i-j is equal to $D_{ij,k}^n P_{Lk}$ .

## III. Congestion Solving Scheme

The control objective of the congestion management scheme is to minimize the generation variation between the original contract and adjusted amount. When the congestion occurs, first the network control is used. If it can eliminate the congestion, no other control is needed. If not, check whether load management is available or not. If it is not available, the minimum generator re-dispatching by using generator contribution factor is chosen. If load management is available, the generator re-dispatching and load management are combined based on the generator and load contribution information. For the generator re-dispatching, decrease the output of the most congestion-contributing generators and increase the output of the least congestion-contributing generators to balance the load and supply. Then run the power flow to check the result. If the congestion is solved and no other line congestion or voltage violation exists, control command will be issued. If not, go to the associated generator re-dispatching and load management analysis and check with power flow untill solving the congestion problem. The flow chart of the control scheme is shown in Fig. 1.

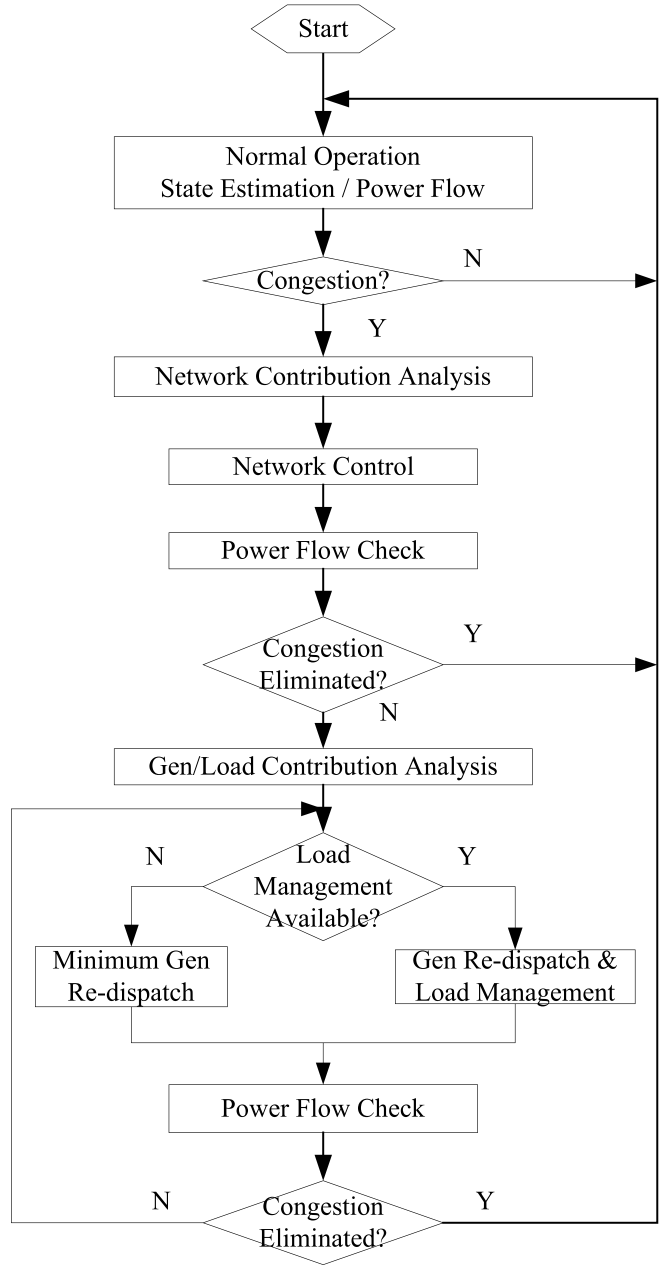


Fig. 1. Flow chart of Congestion Solving Scheme

## IV. Numerical Results

Given a modified IEEE 14-bus system, the load and generation are modified (their values are given in Table I), and transformer branches 5-6, 4-7, 4-9 are changed into transmission lines (with reactance being the same) from the standard IEEE 14 bus system. There is power wheeling from the Area A to the Area B through the tie-lines 5-6, 4-7, 4-9.

Fig. 2 gives the modified IEEE 14-bus system configuration. Table I gives the base flow condition. Columns 2 and 3 are real and reactive load at each bus. Column 4 is shunt capacitance at each bus. Column 5 is generator output. Column 6 is bus voltage magnitude in p.u.. Table II gives the real power flow at each tie-line.

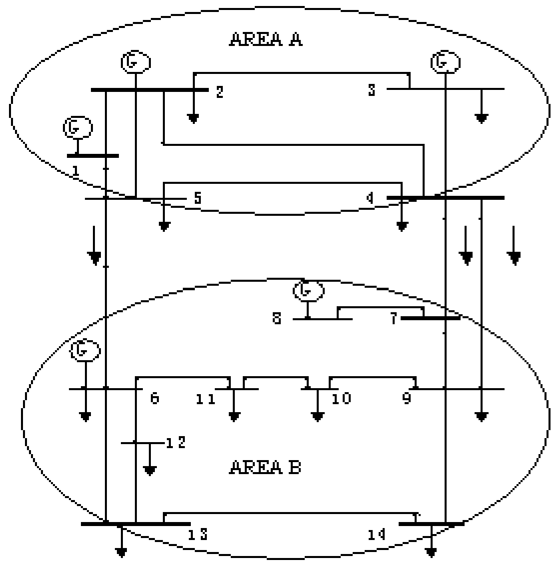


Fig. 2. Modified IEEE 14-bus system

TABLE I
BASE FLOW CONDITION (PD, QD, BS, PG: MVA; V: PU)

| Bus | Pd | Qd | Bs | Pg | V |
|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0. | 232.39 | 1.060 |
| 2 | 21.70 | 12.70 | 0. | 65.00 | 1.045 |
| 3 | 64.20 | 19.00 | 0. | 65.00 | 1.010 |
| 4 | 47.80 | -3.90 | 0. | 0.00 | 0.996 |
| 5 | 7.60 | 1.60 | 0. | 0.00 | 1.001 |
| 6 | 11.20 | 7.50 | 0. | 38.00 | 1.050 |
| 7 | 0.00 | 0.00 | 0. | 0.00 | 1.013 |
| 8 | 0.00 | 0.00 | 0. | 35.00 | 1.060 |
| 9 | 59.50 | 16.60 | 19. | 0.00 | 1.004 |
| 10 | 29.00 | 5.80 | 0. | 0.00 | 0.995 |
| 11 | 23.50 | 1.80 | 0. | 0.00 | 1.010 |
| 12 | 49.10 | 1.60 | 0. | 0.00 | 0.977 |
| 13 | 63.50 | 5.80 | 0. | 0.00 | 0.983 |
| 14 | 44.90 | 5.00 | 0. | 0.00 | 0.950 |

TABLE II
BASE TIE-LINE FLOW (P.U.)

| Branch | Real Line flow |
|---|---|
| 1 (bus 5-6) | 1.1446 |
| 2 (bus 4-7) | 0.6156 |
| 3 (bus 4-9) | 0.4142 |

For security reason, the line 1 flow limit is 1 pu. Network control means need to be taken to decrease the power flow at line 1. Here we assume two network control means, inserting Thyristor Controlled Series Capacitor (TCSC) at each line (assume we have a TCSC at each line) and switching off the line. By assuming all the lines have a capability of 50% compensation capacity by TCSC except for line 1, we get the flow network contribution factor $N_{f,1}$ to the line 1 flow variance.

TABLE III
FLOW NETWORK CONTRIBUTION FACTORS

| Line | Insertion of TCSC | Switch off line |
|---|---|---|
| 2 | -0.0057 | -0.0168 |
| 3 | -0.0152 | -0.0212 |
| 4 | -1.0415e-004 | -6.2903e-004 |
| 5 | -2.0882e-004 | -4.8527e-004 |
| 6 | -8.2135e-004 | -0.0013 |
| 7 | 8.1889e-004 | 0.0014 |
| 8 | 5.9739e-004 | 9.4764e-004 |
| 9 | -2.0171e-004 | -5.1899e-004 |
| 10 | -1.7820e-004 | -9.1447e-004 |
| 11 | 0.0 | PF diverge |
| 12 | -0.0023 | -0.0128 |
| 13 | -4.6534e-004 | -0.0043 |
| 14 | 1.2026e-004 | 4.6382e-004 |
| 15 | 4.8069e-004 | 0.0011 |
| 16 | 7.3215e-004 | 0.0005 |
| 17 | -0.0032 | -0.0113 |
| 18 | -0.0011 | -0.0042 |
| 19 | -9.4881e-005 | -2.4197e-004 |
| 20 | -7.6601e-004 | -0.0021 |

From (25), when inserting TCSC at each line, $y_k \Delta y_i$ is positive. Negative value of contribution factor represents load flow reducing when the real line flow is in the same direction with defined flow direction. When switching off the line, $y_k \Delta y_i$ is negative. Positive value of contribution factor represents load flow reducing.

From Table III, we know that inserting TCSC at line 3 and switching off line 7 respectively contribute the most to relieving congestion on the line 1. When line 7 is switched off, the real power reduction of line 1 is 0.0235 p.u., which is very close to the power flow result, 0.0202 p.u. When the TCSC compensation is 50% of line 3 reactance, the real power reduction of line 1 is 0.0849 p.u., which is very close to the power flow result, 0.0832 p.u.. We can see that 50% compensation at line 3 is better than switching off line 7 for relieving line 1 congestion. In order to decrease the real power flow at line 1 to 1 p.u., the TCSC compensation capacity should be 68.75% of line 1 reactance.

If we only have 50% compensation capacity and no load management available, we need to re-dispatch the generation. After the 50% compensation at line 3, we have the generator contribution factor table as given in Table IV:

TABLE IV
FLOW GENERATOR CONTRIBUTION FACTOR

| Line | Gen 1 | Gen 2 | Gen 3 | Gen 4 | Gen 5 |
|---|---|---|---|---|---|
| 1 | 0.4060 | 0.2305 | 0 | 0 | 0 |
| 2 | 0.1585 | 0.2045 | 0.1451 | 0 | 0 |
| 3 | 0.1067 | 0.1376 | 0.0977 | 0 | 0 |
| 4 | 0.5984 | 0 | 0 | 0 | 0 |
| 5 | 0.1037 | 0.1733 | 0 | 0 | 0 |
| 6 | 0.2206 | 0.3687 | 0 | 0 | 0 |
| 7 | 0.3844 | 0 | 0 | 0 | 0 |
| 8 | 0.1978 | 0.3305 | 0 | 0 | 0 |
| 9 | 0.0374 | 0.0625 | 0.3609 | 0 | 0 |
| 10 | 0.1360 | 0.0772 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 1.0000 |
| 12 | 0.1585 | 0.2045 | 0.1451 | 0 | 1.0000 |
| 13 | 0.0681 | 0.0878 | 0.0623 | 0 | 0.2567 |
| 14 | 0.0469 | 0.0266 | 0 | 0.1155 | 0 |
| 15 | 0.1201 | 0.0682 | 0 | 0.2957 | 0 |
| 16 | 0.2004 | 0.1137 | 0 | 0.4935 | 0 |
| 17 | 0.0801 | 0.1034 | 0.0734 | 0 | 0.3022 |
| 18 | 0.0118 | 0.0153 | 0.0108 | 0 | 0.0446 |
| 19 | 0.0141 | 0.0080 | 0 | 0.0348 | 0 |
| 20 | 0.0118 | 0.0067 | 0 | 0.0291 | 0 |

From Table IV, we can see that Gen 1 contributes most to the line 1 flow. For relieving line congestion, we choose to decrease the output of the most contributing generator and increase the output of the least contributing generator for the same amount. We simply choose a generator pair, for example, Gen 1 – Gen 4 pair. Let Gen 1 reduces its output by (1.1446-1.0-0.0849)/0.406=0.147 p.u., Gen 4 increases the output by 0.147 p.u. The new line 1 power flow result is 0.9782 p.u., which is very close to the scheduled 1 p.u..

If load management is available, the Load Contribution Factor can be calculated. Then appropriate amount of load management will be chosen. The generator re-dispatching and load management can be combined together to get an optimal solution for relieving congestion.

## V. CONCLUSION

This paper introduces a comprehensive method of congestion management by using network, generator and load contribution information. A congestion-solving scheme is presented. It can be used for load flow re-dispatching, congestion management, overload relieving, etc. Numerical results using a modified IEEE 14-bus system show its benefits and advantages.

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## VII. BIOGRAPHIES

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