

Probabilistic Evaluation of the Effect of Maintenance Parameters on Reliability and Cost

Mohsen Ghavami

Electrical and Computer Engineering Department
Texas A&M University
College Station, TX 77843-3128, USA
ghavami14@neo.tamu.edu

Mladen Kezunovic

Electrical and Computer Engineering Department
Texas A&M University
College Station, TX 77843-3128, USA
kezunov@ece.tamu.edu

Abstract—Preventive maintenance is performed to extend the equipment lifetime or at least the mean time between failures. Cost-effective maintenance scheduling is important due to budget constraints in the current situation where reduction of the operating and capital cost is the focus of the power industry. In order to establish a cost-effective maintenance, quantitative evaluation of maintenance parameters is critical. In this paper, a probabilistic model to achieve cost-effective maintenance strategies is presented. Reliability indices such as mean duration, state probability and visit frequency of each state, are computed using Monte Carlo simulation and demonstrated using a numerical example. Further, cost analysis is performed by computing all associated costs including inspection, maintenance and failure costs based on the reliability indices.

Keywords: State diagrams, Deterioration, Maintenance, Inspection, Monte Carlo simulation

I. INTRODUCTION

The utilities perform regular inspection, planned maintenance at a selected working state of components and on-demand repair or replacement at the failure state of component. They have always utilized maintenance programs to keep their equipment in desirable working condition for as long as it is feasible [1]. Probabilistic maintenance models and reliability centered maintenance have been presented to optimize maintenance and reliability costs [2]-[10]. A risk based approach is proposed for maintenance scheduling of circuit breaker in [5]. This approach is different from the other risk based approaches in the way the risk is calculated. It utilizes the maintenance quantification models developed earlier to quantify the circuit breaker maintenance [12]-[13]. These approaches are working pretty well when there is a continuous monitoring or the inspection rate is so high, which results in a lot of available data about the condition of the component.

In the literature, a state diagram is used to represent the deterioration process of the component [1]-[2]. It is assumed that the remaining time in each state is a random variable exponentially distributed [1]-[2], [4], [13]-[14]. With this assumption, the state diagram can be represented by a Markov process and there are some analytical solutions for this model.

Mohsen Ghavami and Mladen Kezunovic are with the Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX 77843-3128, USA(emails: ghavami14@neo.tamu.edu, kezunov@ece.tamu.edu).

In the literature, it is also analyzed whether these models are realistic or not, especially when there is a non-periodic inspection [3]. In this paper, the inspection/maintenance strategy is evaluated by a proposed model. In most of maintenance strategies, the inspection is non-periodic and increased at the end of life cycle of the component. Also, the inspection intervals are deterministic, and the duration of the inspection is a constant number. The model proposed in this paper follows this kind of maintenance strategy.

This paper is focused on the way the life cycle of the component is implemented. Although representing the life time of the component in several discrete stages according to the deterioration levels is well known concept, this paper is different from the rest in the way the life cycle of the component is simulated in a selected algorithm. Specifically the way the transition between the life time stages and inspection stages has been handled is explored. This model would be suitable particularly in the case of non-periodic inspection strategies. The transition time distribution between deterioration stages are assumed to be exponential, where as the transition from deterioration stage to inspection stage is assumed to be a constant number.

The paper is organized as follows. Section II discusses maintenance models using state diagram and transition rates between the states. In section III, a model is proposed to simulate the life cycle of the component. Cost analysis is discussed in section IV. In section V, a numerical example is presented and solved using Monte Carlo simulation to extract the reliability indices of the probabilistic maintenance model, followed by conclusions in section VI.

II. MAINTENANCE MODELING USING STATE DIAGRAMS

It is a matter of common knowledge that component failures are divided into two categories: either random failures or those arising as a consequence of deterioration. Note that these are state models, not Markov model as there are no assumptions made about the time distributions of the individual transitions [1]. The process of deterioration can be thought of a sequence of deterioration stages shown in Fig. 1. In most applications, considering three deterioration stages such as initial stage (D_1), a minor (D_2) and a major (D_3) deterioration stage, is sufficient [2]. If no maintenance is performed, a new component will run through all the stages, respectively. It is

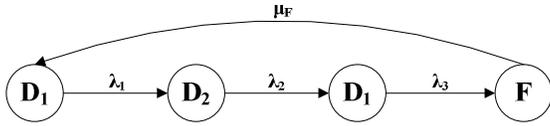


Figure 1. State diagram for modeling the life cycle of the component (without maintenance)

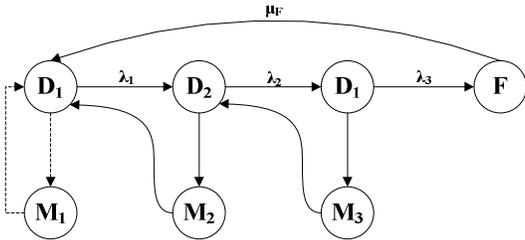


Figure 2. State diagram with adding the maintenance state

reasonable that these stages can be defined by specific signs that appeared in the component because of aging and realized by inspections. It is a good assumption (near reality) that the failure probability of the component is arisen by these consequences of deterioration of stages, and the remaining time of the component in each stage is independent of the time for which the component has been in that stage.

The negative exponential probability distribution is the only one that has the memory less property [15], and it is used to represent the probability of such event. In the real world, most of utilities conduct maintenance actions based on periodic inspection or maybe non-periodic inspection. It means that the state of the system is completely unknown unless inspection is performed [16].

Although more discussions are needed about inspection models, the previous model shown in Fig. 2 can be improved to the model shown in Fig. 3 which includes inspection states. It is shown that there is an inspection state instead of dotted-line for maintenance in state D_1 . In this model, based on inspection results, two kinds of maintenance, M_2 (minor maintenance) or M_3 (major maintenance), can be performed, or the component will be left without any kind of maintenance if it is in state D_1 . The expected result of all maintenance actions is only improvement to the previous stage [6], [17]-[18]. In some literature, waiting periods after inspections are considered. Also some contingencies where no improvement is achieved, or even some damage is done by maintenance activities are reported [2]. For the sake of simplicity, these cases are not considered in this paper. If one assumes that remaining life time in each state has an exponential probability distribution and the transition rate between states are constant numbers, the state diagram will turn into Markov process. There are some analytical methods to solve this probabilistic model and extract reliability indices such as mean durations, visit frequencies and mean time between failures [15], [19]. Monte Carlo simulation can be used to solve this probabilistic model when the answer is difficult to drive through analytical solutions. Maintenance models with this structure are discussed in the literature [2], [4], [13]-[14], [20].

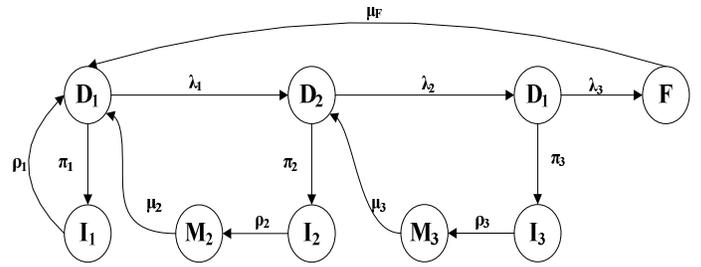


Figure 3. State diagram for modeling the life cycle of the component with inspection/maintenance strategy, the transition duration between states is supposed to be exponentially distributed, so the transition rate is a constant number.

The most important assumption in the model, shown in Fig. 3, is that the maintenance actions are not carried out in a predefined schedule. Based on regular inspections, it can be decided if and what kind of maintenance should be done. The decision after inspection can be either doing nothing (where the condition of the equipment is in deterioration stage D_1) or carrying out specific kind of maintenance denoted by M_2 or M_3 . As seen in Fig. 3, the inspection rates π_1 - π_3 can be equal which means this approach holds for either periodic or non-periodic inspection. It is obvious that the probability of detecting a critical situation at the end of the component's life cycle is increased and returning the component to the previous situation needs more effort. So, it is reasonable that the inspection rate is higher if the equipment is deteriorated more. There is a good discussion and model about non-periodic inspection in [3].

III. EXTRACTING RELIABILITY INDICES USING MONTE CARLO SIMULATION

The goal of this section is to devise a model which follows the maintenance strategy in the real world and afterward solving the model using Monte Carlo simulation. To have compatible results with the maintenance strategy in the real world, two assumptions are considered in this model. First, inspection rates should be increased along with aging of the component. The most of maintenance strategies utilized by utilities have non-periodic inspection rates increasing at the end of life cycle of the component. Second, the remaining time in the inspection state and also inspection duration are deterministic and they cannot be modeled by exponential distributions. Thus, the model is not still Markov process and the answer is hard to derive through analytical solutions. Therefore, the best way to solve the proposed model in this section is Monte Carlo simulation. In general, inspections are performed which leads to three kinds of decisions followed:

- do nothing, if the component is still in initial stage D_1 ;
- Carry out minor maintenance M_2 , if the component is in stage D_2 . This will return the device to stage D_1 ;
- Carry out major maintenance M_3 , if the component is in state D_3 . This will improve the component condition to stage D_2 ;

In order to establish a maintenance strategy including these assumptions, the model shown in Fig. 3 should be improved to the model shown in Fig. 4. The main idea of this model is that the deterioration process and inspection strategy are two parallel processes. It means the deterioration process does not change the next inspection time determined by last inspection. It is more realistic because the state of component is completely unknown if the inspection is not performed [16]. In the model seen in Fig. 3, the inspection rate will change from π_1 to π_2 if the component is deteriorated single step. In the real world, there is a non-periodic inspection which means π_1 is not equal to π_2 , and the next inspection time is determined at the last inspection.

The main goal of this section is to develop an algorithm to simulate the life cycle of a component in a probabilistic model and find out the reliability indices by using the Monte Carlo simulation. To understand the model and the proposed algorithm to solve that, some iteration is followed. The parameters in this model are:

- ζ inspection intervals [years];
- η inspection duration [years];
- μ repair rate [1/years];
- λ deterioration rate [1/years];

Suppose the component is at state $D_x \equiv D_1$ at time $t_0=0$ and it will transit either to state $D_x \equiv D_2$, or inspection state I (there is the same story for the other situations; it means in each step, it transits either to the next deterioration stages or to the inspection state). The simulation of transition from state D_1 to state D_2 can be modeled with a random number generated by an exponential distributed random number generator with rate λ_{12} , denoted by d_{12} . For transition from State $D_x \equiv D_1$ to inspection state I, it is a constant number equal to the inspection interval ζ_1 for the first inspection. Similarly in Fig. 3, π_1 is the inspection rate and $1/\pi_1$ is equal to inspection period, but here the inspection intervals are not exponentially distributed random variables. Thus, the leaving time of the state $D_x \equiv D_1$, is either $(t_0 + d_{12})$ or $(t_0 + \zeta_1)$. As seen in the Fig. 4, the inspection interval denoted by ζ_x , which is similar to the inverse of the transition rate to inspection states in Fig. 3, is varying (it can be equal to ζ_1 , ζ_2 or ζ_3) and it is determined at the last inspection period.

Assume that $(t_0+d_{12}) > (t_0+\zeta_1)$, so first there is a transition from state $D_x \equiv D_1$ to inspection state I. This inspection will show us that the component is still in state $D_x \equiv D_1$ and according to the maintenance strategy, nothing will be performed and the system will return to the state $D_x \equiv D_1$. Approximately, the inspection duration is neglected in comparison to inspection intervals but it can be considered, which is equal to η_1 (inspection duration). Thus, the component is returned to state $D_x \equiv D_1$ at time $(t_0+\zeta_1+\eta_1)$. The next transition can be either to state $D_x \equiv D_2$ or inspection state as before. The most important point in this algorithm is that another random number for transition time from state $D_x \equiv D_1$ to state $D_x \equiv D_2$ has not been generated because the time of deterioration does not change when inspection is performed (inspection does not make any improvement). Therefore, the next transition time

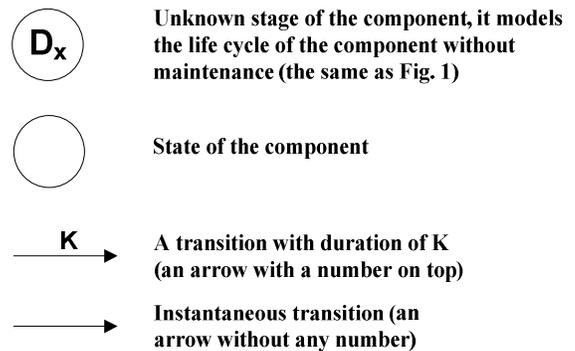
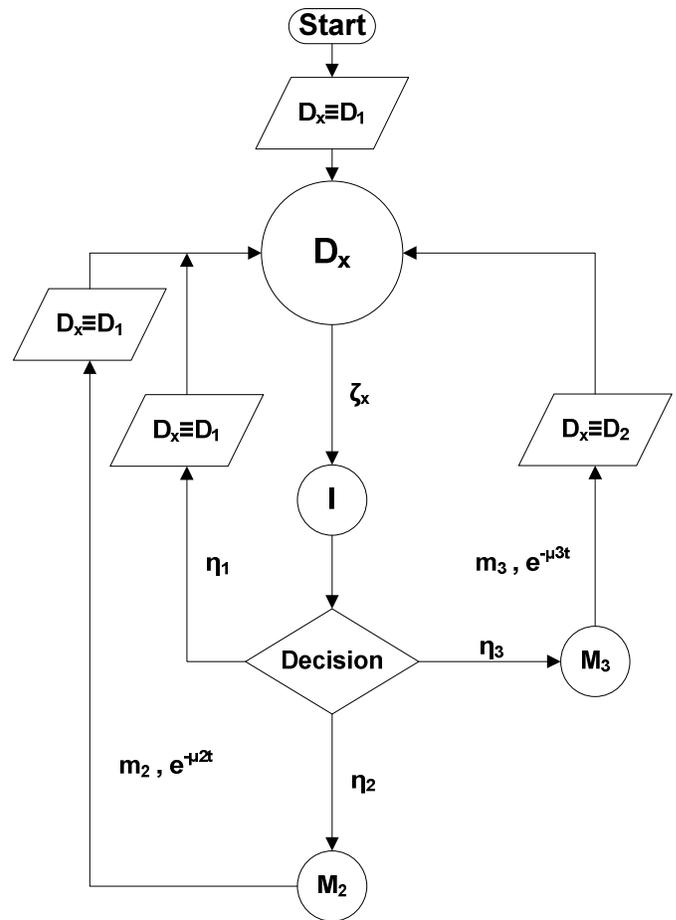


Figure 4. A model to simulate the life cycle of the component with inspection/maintenance strategy. This model is solved by Monte Carlo simulation.

from state $D_x \equiv D_1$ to state $D_x \equiv D_2$ will be (t_0+d_{12}) . This is the main difference between this model and conventional Markov process based model shown in Fig. 3. In that model, after returning from inspection state I, a new number will be regenerated for the deterioration time from State D_1 to state D_2 which is not compatible with the situation in the reality.

Therefore, the next leaving time of state $D_x \equiv D_1$ is either (t_0+d_{12}) or $(t_0+\zeta_1+\eta_1+\zeta_1)$.

Suppose that $(t_0+d_{12}) < (t_0+\zeta_1)$. It means that the deterioration time from state $D_x \equiv D_1$ to state $D_x \equiv D_2$ is less than the inspection time. At time (t_0+d_{12}) , the component will deteriorate to state $D_x \equiv D_2$; afterward there are two possible ways to leave the state $D_x \equiv D_2$. Obviously, it can transit either to state $D_x \equiv D_3$, or to inspection state. The time of transition from state $D_x \equiv D_2$ to state $D_x \equiv D_3$ can be obtained by a random number generated by an exponentially distributed random number generator with rate λ_2 , denoted by d_{23} . As the deterioration transition from state $D_x \equiv D_1$ to state $D_x \equiv D_2$ is not predetermined (it is only a probabilistic model), the next inspection time will not change, which means the next inspection time is still $(t_0+\zeta_1)$ as before. As it is mentioned already, this is the main difference between this algorithm for Monte Carlo simulation and the other algorithms based on Markov process model. In the Markov process models, a new random number will be generated for the next inspection time in this case. Therefore, there are two possible transition time, $(t_0+d_{12}+d_{23})$ and $(t_0+\zeta_1)$.

If $(t_0+d_{12}+d_{23}) > (t_0+\zeta_1)$, it will transit to inspection state where it will be revealed that the component condition is in state $D_x \equiv D_2$, and the specific kind of maintenance M_2 is required. Maintenance is done immediately after inspection and the component's condition will return to the state $D_x \equiv D_1$. The maintenance action duration can be neglected in comparison with the inspection intervals, but it is an exponential random number with rate μ_2 , denoted by m_2 . At time $(t_0+\zeta_1+\eta_2+m_2)$, the condition of the component is "repaired as new" and returns to state $D_x \equiv D_1$. To continue the modeling of the life cycle of the component, the new numbers for transition times have to be derived using random number generators as before. Thus, there will be two numbers; $(t_0+\zeta_1+\eta_2+m_2+d_{12}^{new})$ for transition to state $D_x \equiv D_2$, and $(t_0+\zeta_1+\eta_2+m_2+\zeta_1)$ for transition to inspection state.

Returning to the first assumption in the last paragraph, if $(t_0+d_{12}+d_{23}) < (t_0+\zeta_1)$, it will transit to state $D_x \equiv D_3$. Another number for transition from state D_3 to failure state F has to be derived denoted by d_{3F} . If $(t_0+d_{12}+d_{23}+d_{3F}) > (t_0+\zeta_1)$, it leads to detection of the component condition which is in state $D_x \equiv D_3$. After inspection, it will be decided to perform the major maintenance denoted by M_3 . This kind of maintenance will return the component condition single step back. Finally at time $(t_0+\zeta_1+\eta_3+m_3)$, it returns to state $D_x \equiv D_2$. Be careful that for the next iteration, the inspection interval would be ζ_2 . It could be $(t_0+d_{12}+d_{23}+d_{3F}) < (t_0+\zeta_1)$ which results in failure. It means that after the repair or replacement duration (an exponential random number with rate μ_F) the component condition will return to the state "good as new" denoted by $D_x \equiv D_1$.

In the considerations above, we tried to clarify how the model can be solved using numerical methods. Monte Carlo simulation and reliability indices are explained in detail in [15].

IV. COST ANALYSIS

In this section, the total cost of maintenance/inspection strategy is calculated using some reliability indices derived in the previous section. The total cost includes inspection cost, maintenance cost and failure cost per year. The most important reliability indices which are used to calculate the cost are the

visit frequency and mean duration for inspection, maintenance and failure. These notations are followed:

C_T =total expected annual cost

C_F =the cost of repair or replacement paid after failure

C_{Mx} =the cost of maintenance action type x (M_2 or M_3)

C_I =the cost of inspection

$C_T = C_F$ (frequency of failure state) + C_{M2} (frequency of state M_2) + C_{M3} (frequency of state M_3) + C_I (frequency of state I)

The cost paid after the failure of the component may not include only the repair or replacement cost, but also the cost of event consequences and damages to the entire system should be involved if the supply is interrupted by that failure. There is the same scenario for the maintenance and inspection. Maybe, the cost is needed to take the component out of service for maintenance or inspection. At these situations, the mean duration time of component being at each state is critical and the time can contribute to the cost of that state. In some maintenance strategies, there is some waiting period before it is suitable time for doing maintenance or inspection to reduce the costs. If the duration of these periods are not comparable to inspection intervals, it can be neglected, and that is why it is not mentioned in the model shown in the previous section.

Another element in the cost estimation is the visit frequency, which is calculated based on the maintenance strategy. In Markov process, the visit frequency of state j is the frequency of encountering state j from the other states. In the model in Fig. 3, all the times, when there is a transition from state D_1 to state I, are counted in the visit frequency of state D_1 . Although there is a transition to inspection state, the component is still in deteriorating stage D_1 . So, it should not be considered in the visit frequency and mean duration of state D_1 [3]. In the model shown in Fig. 4, there is the same scenario to figure out the reliability indices using Monte Carlo simulation. Therefore, this point has to be considered to calculate real visit frequency and also the mean duration time.

V. NUMERICAL EXAMPLE

This section presents, a numerical example based on the ideas in the past two sections. The input data is referred to in [2]. The data is obtained from the analysis of a number of 230 KV air-blast circuit breakers with a total operating history of about 3000 years. For the life cycle of this component, three stages are considered; initial stage (D_1), minor deterioration stage (D_2) and major deterioration stage (D_3). If there is no maintenance, these data are obtained:

Mean duration in stage $D_1 = 3$ years;

Mean duration in stage $D_2 = 3.5$ years;

Mean duration in stage $D_3 = 2$ years;

Life time without maintenance = $3+3.5+2 = 8.5$ years;

For cost analysis, some information related to the cost of different activities needs to be known. For example [2]:

The average cost of inspection = \$200;

Table 1
Maintenance strategy parameters

| Work | Interval | Mean duration |
|------------------------------------|----------|---------------|
| Inspection if $D_x \equiv D_1$ | 2 year | 1 day |
| Inspection if $D_x \equiv D_{2,3}$ | 1 year | 1 day |
| Minor maintenance M_2 | — | 1 day |
| Major maintenance M_3 | — | 2 day |
| Repair/replacement | — | 1 month |

Table 2
Model parameters

| parameter | Value [1/year] | Parameter r | Value [year] |
|-------------|----------------|----------------|--------------|
| λ_1 | 1/3=0.33 | ζ_1 | 2 |
| λ_2 | 1/3.5=0.29 | ζ_2 | 1 |
| λ_3 | 1/2=0.5 | ζ_3 | 1 |
| μ_2 | 360 | η_1 | 1/360=0.003 |
| μ_3 | 180 | η_2 | 1/360=0.003 |
| μ_F | 12 | η_3 | 1/360=0.003 |

The average cost of minor maintenance $M_2 = \$1200$;

The average cost of major maintenance $M_3 = \$14400$;

The average cost of failure = \$144000;

The summary of maintenance strategy followed by the utility from this example is shown in Table I. Neither the waiting periods, nor the probabilities for reaching to another state except target state are considered.

These are all the information used as input data for this model. As shown in Fig. 4, there are some values such as η , ζ , λ and μ , which have to be calculated based on the input data. The transition rate is the inverse of mean duration time in each state and also the maintenance duration is considered as an exponential distributed function of time. Model parameters, shown in Table II, can be easily computed based on the maintenance strategy parameters. After solving the model shown in Fig. 4 using Monte Carlo simulation, the extracted reliability indices are gathered in Table III. In section III, the method of solving the model is explained and it is discussed that these results are more realistic and more proper for cost analysis.

For cost analysis, discussed in section IV, the results are mentioned below.

$$C_1 (\text{frequency of state I}) = 200 \times 0.522 = \$104$$

The annual expected inspection cost = \$104

$$C_T = C_F (\text{frequency of failure state}) + C_{M_2} (\text{frequency of state } M_2) + C_{M_3} (\text{frequency of state } M_3) + C_1 (\text{frequency of state I})$$

Table 3
Reliability indices

| State | State probability [%] | Frequency [1/year] | Mean duration [year] |
|------------------|-----------------------|--------------------|----------------------|
| I | 0.157 | 0.522 | 0.003 |
| M_2 | 0.051 | 0.171 | 0.003 |
| M_3 | 0.026 | 0.042 | 0.006 |
| $D_x \equiv F$ | 0.323 | 0.039 | 0.083 |
| $D_x \equiv D_1$ | 62.43 | 0.208 | 3 |
| $D_x \equiv D_2$ | 29.82 | 0.250 | 1.193 |
| $D_x \equiv D_3$ | 7.193 | 0.082 | 0.877 |

$$C_{M_2} (\text{frequency of state } M_2) + C_{M_3} (\text{frequency of state } M_3) = 1200 \times 0.171 + 14400 \times 0.042 = \$810$$

The annual expected maintenance cost = \$810

$$C_F (\text{frequency of failure state}) = 144000 \times 0.039 = \$5616$$

The annual expected failure cost = \$5616

Finally, by summation of all cost terms, the total cost can be calculated.

The total expected cost per year = \$6530.

If there is no maintenance, the total yearly costs are about $144000/8.5 \approx \$17000$. The comparisons indicate the beneficial effects of maintenance strategy. The only parameter which can change in maintenance strategy devised by utility is inspection rates (inspection intervals) because the inspection, maintenance and repair duration cannot change easily. Utilities can find the optimum inspection rate using sensitivity analysis which means evaluating the effect of changing input parameters (which is inspection rate in this model) on any of the output variables. The most economical policy may not necessarily be the policy selected by the electric utility managers today. It depends on the other criteria defined by electric utility managers. Sensitivity analysis for this model can show how the policy is economical, and it can be considered in decision making.

VI. CONCLUSION

In this paper, a quantitative connection between maintenance parameters and reliability indices is studied using state diagrams in maintenance modeling. The life cycle of the component is simulated based on a probabilistic model and reliability indices are extracted by solving this model. In this model, the inspection intervals are deterministic and they can be non-periodic. This has to be solved using numerical methods such as Monte Carlo simulation. It has been shown that there are some discrepancies with reality in traditional deterioration stages model based on Markov process, especially when there is a non-periodic inspection. All associated costs are discussed along with a numerical example. The electric utility managers can optimize their costs using sensitivity analysis in this model with varying the maintenance parameters and observing the final costs.

In this paper, the off-line inspection strategy is discussed, and the other inspection strategies remain to be explored. Obviously, a model cannot describe all the maintenance/inspection strategies, but it can be a good representation of reality in some cases. The mean duration of failure, inspection and maintenance are not mentioned in the cost analysis, but it is mentioned that the cost associated with those may not be only the action cost, and in the system level, the event consequence and damages to the other parts of the system may also have a significant effect on increased costs. Studies in these areas are in progress, and the solution is the focus of future work.

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