

# Spectral Interpolation for Frequency Measurement at Off-Nominal Frequencies

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**Abstract**—Frequency is one of the most vital parameters used in power system monitoring, protection, and control. The commonly used frequency domain methods for frequency estimation are based on the compensation of DFT spectrum. Some accurate frequency estimation methods utilize the linearization of local DFT spectrum, such as interpolated DFT, primarily aiming at higher estimation accuracy of local maxima of signal spectrum. The accuracy of these methods is questionable due to the distortion from spectrum leakage and picket fence effects. An algorithm that estimates the frequency of quasi-steady state sinusoidal signals is presented in this paper. The proposed algorithm produces accurate frequency estimation by calculating DTFT parameters using nonlinear interpolation method on DFT spectrum. IIR filter is utilized for signal conditioning. To expedite iteration process, autocorrelation function of the original signal is analyzed. Test results show that the proposed algorithm achieves higher accuracy than the requirements from IEEE standard C37.118.1a-2014.

**Index Terms**—Autocorrelation function, discrete Fourier transform, frequency estimation, power system measurements, spectral analysis

## I. INTRODUCTION

Frequency estimation is crucial to maintaining a more secure, resilient and adaptable power grid. Accurate determination of frequency of the fundamental phasor is the basis of many smart grid applications and control schemes [1-3]. Facilitated by powerful phasor estimation algorithms and GPS-synchronized hardware platforms, phasor measurement units (PMUs) and PMU-enabled intelligent electronic devices (IEDs) have become the reliable data sources of modern smart grid. Those devices also benefit from and depend on accurate fundamental frequency estimation to calculate amplitude and phase angle.[4,5] Recently, researchers have begun constructing a cost-efficient measurement network that enables synchronized wide-area observation of power system frequency, using the aforementioned IEDs.[3,6]

In the majority of operating time, the power grid is in steady state, where the frequency may be off-nominal but re-

mains constant; or in a quasi-steady state, such as during the inter-area oscillations [7], where the frequency is slowly changing, but can be considered fixed in one observation window. Therefore, frequency estimation in such scenarios is of particular importance and should maintain high accuracy and consistency. Within the wide spectrum of frequency estimation methods, zero-crossing [8] is the existing method suggested in IEEE standard. This approach can cause large error in the presence of noise and DC offset. Another widely used method calculates frequency from the derivative of phase angle measurements.[9,10] This method essentially performs averaging of instantaneous frequencies over the time instants associated with phase angle measurements. This approach exhibits problems in timestamp alignment, as well as in the error magnification from derivative calculation.[11] More accurate methods attempt to model frequency and noise parameters directly in the signal model. The examples are the Newton-type method [12,13], and the Kalman filter method [14]. These methods are often time-consuming and may raise stability and convergence issues.

A large proportion of literature is focused on interpolated DFT (IpDFT), which was first introduced in [15], as an effort to estimate the frequency using off-nominal frequency static input. Usually, linearization is used to approximate DTFT, which is the continuous signal spectrum. Typically, three DFT harmonics are used to estimate the local maximum of DTFT, which corresponds to the frequency offset determined by off-nominal frequency input. To alleviate the influence from negative frequency leakage, raised cosine windows with lower side lobe levels, such as Hamming or Hann windows, are often used instead of rectangular window. An enhanced interpolated DFT method was proposed in [16], in which the three DFT harmonics are refined to alleviate the tails produced by other frequency components before further estimation. Recently, a frequency estimation algorithm that iteratively estimates the peak of signal amplitude spectrum was proposed [17].The algorithm produces higher accuracy than linearization-based IpDFT, but overlooks the fact that due to complex number superposition, the peak of amplitude spectrum does

not necessarily corresponds to frequency offset. Besides, unknown amplitude and phase angle further complicate the non-linear optimization problem and can potentially raise convergence issues.

In this paper, an algorithm that estimates the frequency offset using the interpolation on the spectrum of signal autocorrelation function is proposed. The rest of the paper is organized as follows. In section II, discrete input Fourier transforms are revisited, and the theoretical background is introduced. Frequency estimation algorithm using spectral interpolation of autocorrelation function is proposed in section III. Simulations are performed and the results are analyzed in section IV. Conclusions are given in section V.

## II. DISCRETE INPUT FOURIER TRANSFORMS BACKGROUND

### A. Continuous Spectrum of Rectangular Window

Rectangular window is extensively discussed for its simplicity, also because various commonly used window functions are derived from the sidelobe cancellation of rectangular window spectrum. [18]

The continuous spectrum, presented by discrete-time Fourier transform (DTFT), of rectangular window is calculated in (1),

$$\begin{aligned} X_{Rect}(\omega) &= \sum_{n=0}^{N-1} e^{-i\omega n} = \frac{\sin \frac{1}{2}\omega N}{\sin \frac{1}{2}\omega} e^{-i\frac{1}{2}\omega(N-1)} \\ &= N \cdot \text{asinc}_N(\omega) e^{-i\frac{1}{2}\omega(N-1)} \end{aligned} \quad (1)$$

where  $\text{asinc}_N(\omega)$  denotes the aliased sinc function.

It is known that discrete Fourier transform (DFT) is the sampling of DTFT in one period of  $2\pi$  at normalized frequencies  $k\omega_0 = \frac{2\pi k}{N} = \frac{2\pi k f_0}{f_{sam}}$ , where  $f_{sam}$  is the sampling frequency,  $N$  is the number of samples in a window, and  $f_0$  is system nominal frequency.[19] As shown in Fig. 1, where DTFT is depicted as dashed line, sampling  $X_{Rect}(\omega)$  at points  $\omega = k\omega_0$ , shown as the black dots, we acquire the DFT of rectangular window:

$$\begin{aligned} X_{Rect}(k\omega_0) &= X_{Rect}(\omega)|_{\omega=k\omega_0} \\ &= N \cdot \text{asinc}_N(k\omega_0) e^{-i\frac{1}{2}k\omega_0(N-1)} \end{aligned} \quad (2)$$

It should be noticed that one equivalent interpretation of the relationship between DTFT and DFT is, DTFT is the nonlinear interpolation of DFT samples using  $\text{asinc}(\cdot)$  function.

### B. DFT Spectrum Interpolation Method Revisited

In signal processing, it is well known that a single-tone sinusoidal signal can be expressed as the combination of two complex signals shown in (3):

$$x(t) = A \cos(\Omega_x t + \phi_x) = A \frac{e^{j(\Omega_x t + \phi_x)} + e^{-j(\Omega_x t + \phi_x)}}{2} \quad (3)$$

The DTFT of single-tone input  $x(t)$ , or  $X_{ST}(\omega)$ , can be achieved by (4):

$$\begin{aligned} X_{ST}(\omega) &= \sum_{n=-\infty}^{+\infty} x_{ST}(t)|_{t=\frac{n}{f_{sam}}} \cdot e^{-i\omega n} \\ &= \frac{A}{2} X_{Rect}(\omega - \omega_x) e^{j\phi_x} + \frac{A}{2} X_{Rect}(\omega + \omega_x) e^{-j\phi_x} \end{aligned} \quad (4)$$

where  $\Omega_x$  denotes the unknown signal frequency in rad/s,  $\omega_x$  represents unknown signal frequency normalized by  $f_{sam}$ .

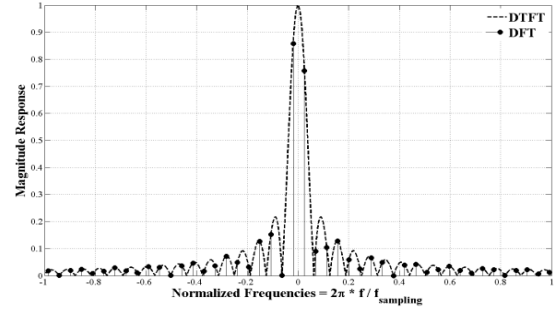


Figure 1. DTFT and DFT of Rectangular Window

Shown in Fig. 2, blue dashed line is spectrum with positive frequency offset,  $X_{Rect}(\omega - \omega_x)$ , and the red dotted line shows the spectrum with negative frequency offset,  $X_{Rect}(\omega + \omega_x)$ , the black solid line shows the DTFT of original signal and, DFT is shown in black dots.

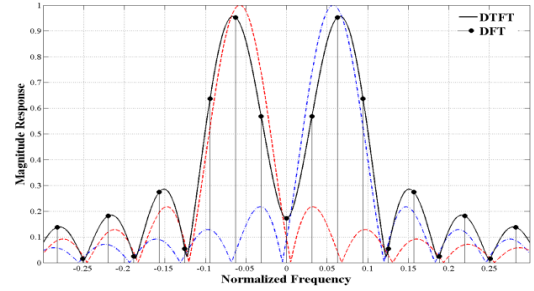


Figure 2. DTFT and DFT of Single Tone Sinusoidal Input

Interpolated DFT (IpDFT) method typically uses three DFT samples to estimate the  $x$ -coordinate of the maxima of linearized DTFT curve. It is worth noting that, however, the maxima of DTFT curve (black solid line) will generally deviate from the actual frequency offset, denoted by the maxima of  $\text{asinc}$  functions (red, blue lines). The deviation is caused by leakage effect from high sidelobe level. Accordingly, window functions with reduced sidelobe levels, such as Hamming and Hamm windows, are commonly used. To some extent, such solution reduces leakage effect. However, if higher estimation result is desired, a higher order window function, which requires data window longer than four cycles, needs to be applied.

Moreover, nonlinear curve fitting techniques [20] can be used to estimate  $\omega_x$ , since the structure of signal model is practically known as (4). Given the initial value, which is set to be normalized nominal frequency  $\omega_x^{(0)} = \frac{2\pi f_0}{f_{sam}}$ , Levenberg-Marquardt algorithm [13] can be leveraged for estimation of unknown parameters  $A$ ,  $\phi_x$ , and  $\omega_x$ , with fast and accurate convergence.

## III. FREQUENCY ESTIMATION USING SPECTRAL INTERPOLATION OF AUTOCORRELATION FUNCTION

A single-tone sinusoidal signal can be described using three parameters: amplitude, frequency (deviation), and phase angle. Usually all three parameters are unknown, and frequen-

cy estimation depends on parameters relevant to amplitude and phase angle estimation, for instance, time-domain methods [9-11]. This paper proposed an algorithm that only preserves the frequency information of a sinusoidal wave.

#### A. Autocorrelation Function of A Sinusoidal Signal

The autocorrelation function (ACF) of a signal is widely used in time-series analysis, and it is an indicator of how a signal resembles its time-shift copy.[21] By doing so, patterns, such as periodicity of the original signal, can be revealed. ACF at lag  $\tau$  is theoretically expressed in (5)

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)} = \frac{\mathcal{E}[(x(t)-\bar{x})(x(t+\tau)-\bar{x})]}{\sqrt{\mathcal{E}[x(t)-\bar{x}]^2 \cdot \mathcal{E}[x(t+\tau)-\bar{x}]^2}} \quad (5)$$

where  $\mathcal{E}(\ast)$  denotes the ensemble average,  $\bar{x}$  is the sample mean, and as can be easily seen,  $\rho(\tau) \leq \rho(0) \equiv 1$ .

Assuming stationarity and ergodicity, which is usually the case, the ensemble average can be estimated using the data from one single experiment, in which auto-covariance  $\gamma(\tau)$  is calculated using (6):

$$\begin{aligned} \gamma(\tau) &= \mathcal{E}[(x(t) - \bar{x})(x(t + \tau) - \bar{x})] \\ &= \mathcal{E}[x(t)x(t + \tau)] - \bar{x}^2 \end{aligned} \quad (6)$$

where  $\mathcal{E}[x(t)x(t + \tau)]$  is calculated as shown in (7):

$$\begin{aligned} \mathcal{E}[x(t)x(t + \tau)] &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)x(t + \tau) dt \quad (7) \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T A^2 \cos(\Omega t + \phi) \cos(\Omega t + \Omega \tau + \phi) dt \\ &= \lim_{T \rightarrow \infty} \frac{A^2}{T} \int_0^T [\cos \Omega \tau + \cos(2\Omega t + \Omega \tau + 2\phi)] dt \\ &= \lim_{T \rightarrow \infty} \frac{A^2}{2T} [\sin(2\Omega t + \Omega \tau + 2\phi) - \sin(\Omega \tau + 2\phi)] + \frac{A^2}{2} \cos \Omega \tau \end{aligned}$$

Note that in (7), the limit term will be suppressed since both sine terms have finite boundaries, therefore, the ACF:

$$\rho(\tau) = \frac{\frac{A^2}{2} \cos \Omega \tau - \bar{x}^2}{\frac{A^2}{2} - \bar{x}^2} \quad (8)$$

As can be seen from (8), the ACF of an off-nominal frequency sinusoidal wave is a sinusoidal wave with unit amplitude, zero phase angle, and the same frequency as the original signal. As shown in Fig. 3, a 55Hz off-nominal sinusoid with initial phase angle  $\pi/4$ , and its ACF, which is a pure cosine wave at the same frequency. Therefore, by taking the ACF of off-nominal frequency signal, only frequency information of original signal is extracted.

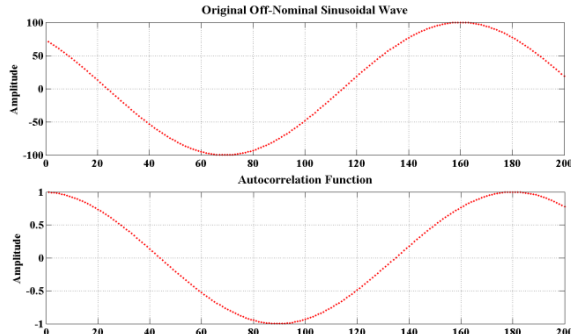


Figure 3. DTFT and DFT of Rectangular Window

#### B. Frequency Estimation

As shown in Section II, the DTFT of the ACF of an off-nominal sinusoidal signal takes the form of:

$$\begin{aligned} X_{ACF}(\omega) &= X_{Rect}(\omega - \omega_x) + X_{Rect}(\omega + \omega_x) \\ &= N \cdot \text{asinc}_N(\omega - \omega_x) e^{-i\frac{1}{2}(\omega - \omega_x)(N-1)} \\ &\quad + N \cdot \text{asinc}_N(\omega + \omega_x) e^{-i\frac{1}{2}(\omega + \omega_x)(N-1)} \end{aligned} \quad (4)$$

where  $\omega_x = \frac{2\pi f_x}{f_{sam}} = \frac{\Omega_x}{f_{sam}}$ .

Per discussion in Section A, the frequency estimation can be eventually equated with the frequency estimation of its ACF, which has the general form of  $\rho(\tau) = \cos(\Omega_x \tau)$ . Therefore, nonlinear curve fitting method, as discussed in Section II.B, can be developed to estimate off-nominal frequency  $\Omega_x$ . Note that theoretically, signal phase angle will always be zero after taking the ACF. Therefore, rigorous filtering techniques can be applied, even though such conditioning of original signal may cause nonlinear group delay, which is always avoided in other methods.

### IV. IMPLEMENTATION AND SIMULATION RESULTS

#### A. Frequency Estimation Procedure

The raw input signal for PMUs and other IEDs features noise and harmonics, and therefore, should be conditioned before feeding into the algorithm. Infinite Impulse Response (IIR) filters are used, because 1) phase distortion from IIR nonlinear group delay will not affect estimation after taking the ACF, and 2) IIR filters can be implemented with lower order than Finite Impulse Response (FIR) filters to achieve comparable performances, hence shorter window length. The procedure of using proposed algorithm to estimate frequency is depicted in Fig. 4.

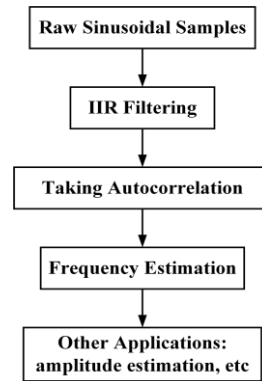


Figure 4. Frequency Estimation Procedure

#### B. Hardware Implementation

The frequency estimation algorithm, as well as data acquisition code is implemented in National Instruments CompactRIO™ (cRIO) device coded in LabVIEW™, as shown in Fig. 5. The data acquisition program is coded on FPGA chip so that speed and stability is guaranteed. Data conditioning, processing and computing is conducted on host computer embedded in the chassis, which runs LabVIEW™ Real-Time system. Since FPGA is pure hardware-based, the execution of program on FPGA is faster than signal processing code on

host computer, which is software-based. Therefore, sampled raw data are transmitted through first-in-first-out (FIFO) buffer between FPGA and host computer to allow lossless data transfer. The hardware system is located at Power System Control & Protection Lab on Texas A&M University campus.



Figure 5. Hardware Implementation

### C. MATLAB Simulation and Result Analysis

Three scenarios are considered in algorithm testing: pure single-tone sinusoidal input, sinusoidal input with harmonic injection, and slow-varying oscillation input.

#### 1) Single-Tone Sinusoidal Input:

Single-tone sinusoids at frequencies ranging from 55Hz to 65Hz with 1Hz increment are utilized as test signals in this scenario. The initial value for frequency iteration is normalized nominal frequency. Test results shows that, under theoretical signal input, the proposed algorithm can achieve zero estimation error.

#### 2) Sinusoidal Input with Harmonic Injection:

Single harmonic from 2<sup>nd</sup> to 50<sup>th</sup> order is injected into sinusoids at frequencies within 2Hz range around nominal frequency. The total harmonic distortion (THD%) level is 1%, referencing the IEEE standard for PMU testing.[22] Type I Chebyshev filter is used to achieve minimum flat frequency response in stopband. As shown in Fig. 6, frequency estimation errors in all test cases are smaller than 0.16mHz, which is at least 10 times higher than IEEE standard requirement (5mHz).

#### 3) Slow-Varying Oscillation Input

Power system is subject to low-frequency oscillations typically within 2Hz.[23] In this test, the oscillation is modeled as a pure amplitude modulation signal.[24] Cases with oscillation frequency ranging from 0Hz (no oscillation) to 2Hz, frequency deviation within  $\pm 2$ Hz from nominal are tested and depicted in Fig. 7. In Fig. 7, results represented by larger red dots indicate that absolute frequency errors are within 30mHz, which is 10 times better than [22], otherwise they are shown as smaller blue dots. As can be seen, the algorithm can withstand oscillations slower than 1Hz and frequency deviation from -1Hz to 2Hz. The proposed method has better accuracy at positive frequency deviation when the mainlobes of off-nominal component are less interfered by the spectral leakage from modulation components.

### D. Hardware Testing and Result Analysis

In this test scenario, National Instrument cRIO device is connected to the power socket on the wall. Single phase voltage is continuously sampled at 5kHz. It is expected that voltage amplitude and frequency are constant in a short estimation window length, and the waveform should be around 60Hz and

contains harmonics. Fast Fourier transform (FFT) is first performed to provide a rough illustration of sampled voltage, with a window length of 500ms so that FFT frequency resolution can be as high as 2Hz. Then frequency is estimated according to Fig. 4.

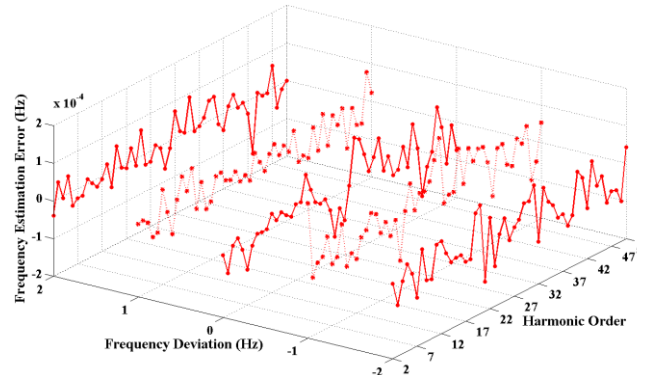


Figure 6. MATLAB Simulation Results for Harmonic Injection Tests

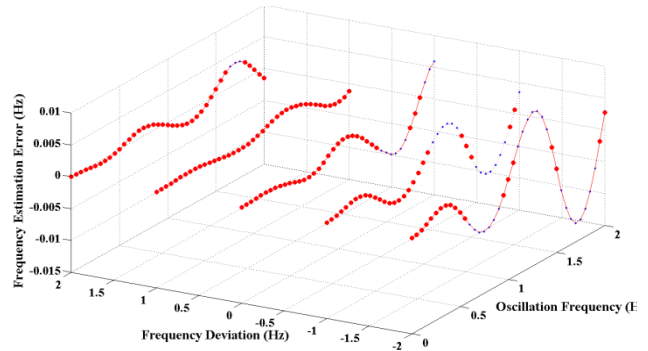


Figure 7. MATLAB Simulation Results for Oscillation Input Tests

Shown in Fig. 8 is the FFT spectral analysis of raw sample from the power socket. It is shown that a wide range of harmonics is presented in the samples, with 5<sup>th</sup> and 7<sup>th</sup> harmonics being the predominant ones. A zoom-in picture of the frequency components around 60Hz shows that the frequencies are spread out around nominal frequency, which is caused by spectral leakage, indicating that the frequency is off-nominal.

Frequency of the signal from 110V power socket was estimated over the course of 240s. In order to show frequency fluctuation, estimation results are reported every 5 seconds. Because of the absence of a reliable frequency reference, frequency information from [25] was used for a qualitative comparison with the calculations from proposed method. The estimation result is shown in Fig. 9.

## V. CONCLUSION

In this paper, a frequency estimation method that uses the spectral interpolation of signal autocorrelation function is proposed. The proposed algorithm aims to calculate frequencies at quasi-steady states: off-nominal with harmonic input, and low frequency amplitude oscillations. The contributions are summarized as follows:

- The proposed frequency estimation method can achieve high accuracy under off-nominal input, even with the pres-

ence of harmonics. It can withstand slow-varying amplitude oscillation under 1Hz.

- Spectral interpolation utilized in frequency estimation improves estimation improves traditional linearized interpolated DFT method.
- Autocorrelation function is used to extract only the frequency information from original signal, thereby increasing estimation efficiency.
- Infinite Impulse Response filters are used to achieve better frequency response performance. The nonlinear group delay caused by IIR filters will not affect frequency estimation after using autocorrelation function.
- The propose method has been implemented in National Instruments hardware and is pending further testing.

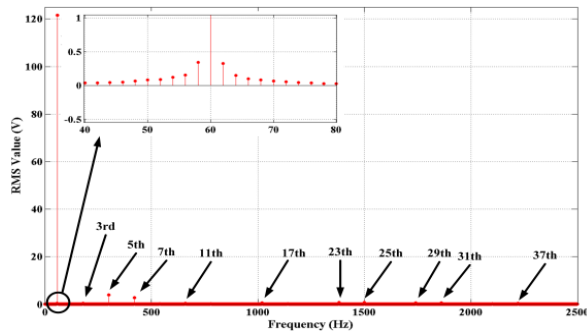


Figure 8. FFT Spectral Analysis of Input Signal

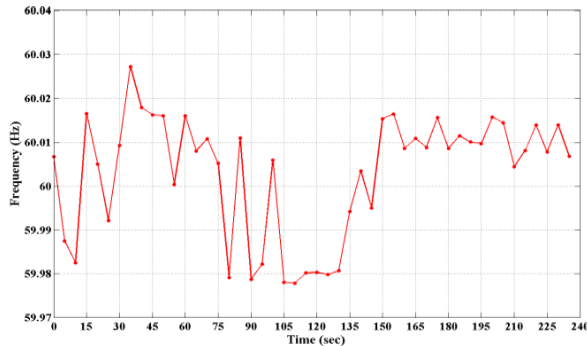


Figure 9. Frequency Estimation on Power Socket

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