A New Approach to PID Controller Design of STATCOM

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Abstract—As a common reactive power compensation equipment in power system, the static synchronous compensator (STATCOM) has played a very important role in the stable operation of power system, which requires fast response and well-designed controller. This paper presents a new approach to proportional-integral-derivative (PID) controller design of STATCOM based on root counting and signature theory with the help of computer-aided calculation. The complete coefficient set of stabilizing PID controllers can be found, which is superior to the traditional trial and error method. The performance specification could be considered within the entire stabilizing set to identify the desired values of coefficient gains of PID controller. Case study is implemented to illustrate and verify the feasibility of the proposed approach.

Index Terms—STATCOM control, PID, model based design, root counting, performance specification, cascading event

I. INTRODUCTION

POWER system security and stability are the top priority when operating the electric power system. All the other targets, such as economic dispatch and optimal load flow, are all subject to the requirements of system security and stability. Power systems are exposed to all kinds of disturbances, which may come from bad weather, human errors, equipment malfunction, natural disasters, etc., and some of these disturbances may cause power service interruption. This situation is getting even worse under new market rules introduced by deregulation causing more stressed operation.

Among the disturbances, cascading outage, especially the large-scale cascading outage, draws special attention since it can cause great economic damage to the power system and devastating impact on people’s life. For example, Northeastern System Blackout in 2003 led to the load loss of 61.8GW, which influenced more than 50 million people [1]. The recent research efforts and solutions are aimed at understanding and finding ways to detect, prevent, and mitigate the cascading events. [2-5] A novel interactive scheme of system/local monitoring and control tools for detection, prevention and mitigation of cascading events was recently introduced [6-13]. This scheme can identify the disturbance information and provide control means for preventing further unfolding of the cascading events and keeping the stability of the power system. Another very important approach in to mitigating the cascading event is trying to take some preventive actions before the disturbances happen.

As we all know, the power system stable operation is supported by many control means, such as exciter control system, reactive power compensation, frequency regulation service, etc. Selecting suitable parameters of the controller will improve the control performance and system ability to resist instabilities. Reactive power control is one of the most critical services. Shortage of reactive power will increase the transmission losses, degrade power transmission capability, decrease load end voltage regulation, and increase the chance for the system outages [14]. With the development of power electronics and the increasing power rating achieved by solid-state devices, the Static Synchronous Compensator (STATCOM) has taken the places of the traditional Thyristor-Controlled Reactors (TCR) and Thyristor-Switched Capacitors (TSR) as the reactive power compensation equipment. Thus the continuous reactive power control with fast response is the main requirement for the STATCOM controller [15].

The control of STATCOM has been discussed in many published papers, which includes state feedback control techniques, pole placement method, etc. [16-18]. The ultimate aim is to determine the optimal parameters for the controller by using genetic algorithm, linear quadratic regulator (LQR) method, etc. However, an important method for PID (Proportional-Integral-Derivative) controllers was reported in [19, 20]. In this paper, the new PID controller design and synthesis approach for STATCOM control is introduced based on the root counting and signature theory [21, 22]. This approach can find the complete set of stabilizing PID controllers in the PID coefficients gain space. Once the entire set is found, it will be very easy to solve the performance attainment problems and meet the desired requirements such as guaranteed gain and phase margins, since the performance results can be evaluated by sweeping the PID parameter space under the given plant.

This paper is organized as follows: Section II introduces the overview of STATCOM modeling. The design and synthesis approach for the PID controller is described in detail in Section III. Section IV presents the implementation
methodology. Simulation cases and comparative results are shown in Section V. Conclusions are given in Section VI.

II. MATHEMATICAL MODEL OF STATCOM
Because of the high dynamic performance, STATCOM has become one of the most effective equipments for reactive power compensation. The compensation does not depend on the common coupling voltage, which makes STATCOM solution attractive due to its advantages: precise and continuous reactive power control with fast response, and minimal interaction with power grid. STATCOM is becoming a predominant new generation devices for flexible AC transmission systems (FACTS).

The mathematical model of STATCOM has been discussed in the literature [14, 15, 17]. Figure 1 shows a typical STATCOM configuration from literature [14]. The DC bus voltage is built up across the DC capacitor. By controlling the firing command for each of the three-phase bridges, the desired voltage across a bridge will be generated and the current through the line impedance $R$ and $L$ is controlled [14].

![Fig. 1. STATCOM system configuration](image)

STATCOM is a Multiple Input Multiple Output (MIMO) system. Thus a multivariable control approach is needed for the STATCOM control design. Although it is not possible to totally decouple the system variables, there is one powerful tool for studying balanced three phase system, which converts the three phase voltages and currents into orthogonal components in a synchronous rotating frame by Park Transform. The MIMO system will be simplified for the decoupling method. The orthogonal components in the rotating frame are referred to as active and reactive components. The proposed approach for PID controller design and synthesis will be applied for the decoupled control variable.

The mathematical expression of the STATCOM system is given in equation (1) to (4) [14]

$$\frac{di_d}{dt} = -\frac{R}{L}i_d + \omega i_q + \frac{1}{L}(V_d - V_{sd})$$  \hspace{1cm} (1)

$$\frac{di_q}{dt} = -\omega i_d - \frac{R}{L}i_q + \frac{1}{L}(V_q - V_{sq})$$ \hspace{1cm} (2)

$$\frac{dV_{dc}}{dt} = -\frac{3(V_{sd}i_d + V_{sq}i_q)}{2CV_{dc}} + \frac{i_d}{C}$$ \hspace{1cm} (3)

$$Q = \frac{3}{2}(V_{sq}i_q - V_{sd}i_d)$$ \hspace{1cm} (4)

where $\omega$ is the angular power frequency, and subscripts $d, q$ represent variables in the rotating coordinate system for the components of direct and quadrature axis, respectively.

Choosing the states $x$, the inputs $u$, and the output $y$ by:

$$x = \begin{bmatrix} i_d \\ i_q \end{bmatrix}, u = \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix}, y = \begin{bmatrix} i_d \end{bmatrix}$$

The equations (1) and (2) can be rewritten to the state space transfer function as the linear system:

$$\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}$$ \hspace{1cm} (5)

where the corresponding coefficient matrices are:

$$A = \begin{bmatrix} -\frac{R}{L} & \omega \\ -\omega & -\frac{R}{L} \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The detailed STATCOM control block diagram is described in literature [17], which includes two control loops in the whole system for the decoupled variable. The new approach will be applied for each variable, which will be discussed later.

III. DESIGN AND SYNTHESIS OF PID CONTROLLER
In this section, we develop the new approach for the PID controller design and synthesis of STATCOM.

A. PID Controller

PID controller is one of the most common controlling devices in the market. Because of its very simple control structure and the linear control methodology, PID control is generically important in many industries and has been widely used in electrical, mechanical, hydraulic, fluidic, and pneumatic systems [21]. It can provide the set point regulation of zeroing error under arbitrary low frequency disturbances, and it owns robust characteristics for those modeling errors. Three term controllers are easier to adjust at the design stage as well as online.

Consider the general feedback system with a PID controller and plant transfer function $G(s)$, which is shown in Figure 2.

![Fig. 2. A feedback system with PID controller](image)

Where $r(t)$ is the reference signal, $u(t)$ is the input signal,
\( y(t) \) is the output, \( C(s) \) is the controller to be designed. For the PID controller, \( C(s) \) will be:

\[
C(s) = k_p + \frac{k_i}{s} + k_ds
\]  

where \( k_p \), \( k_i \) and \( k_d \) are the proportional, integral and derivative gains respectively. In some cases when the error is measured in a noisy environment, a delay part for the transfer function should be considered as:

\[
C(s) = \frac{sk_p + k_i + k_ds^2}{s(1 + sT)}, \quad T > 0
\]  

where \( T \) is usually a small positive value.

The design of PID controller is to determine the values of coefficients \( k_p, k_i, \) and \( k_d \), which could make the controller stabilize the given plant. When the controller is used in industries, the performance of the feedback system under the control of the designed PID controller should also be considered. Those parameters should be adjusted so that acceptable performance, such as the rising time, settling time, gain margin, etc. could be obtained. Many controller design methods, such as a model based approach using state space models, state feedback control, and quadratic optimization [23], fixed order controller design in discrete-time system [24], model free synthesis approach based on time series data [25], etc. have been presented in the past years. The tuning methodologies of the PID controller have been developed over the years based primarily on empirical observations and industrial experience.

B. Fixed Order Controllers Design based on Root Counting

Fixed order controller design based on root counting theory is extended from the classical Hermite-Bieler Theorem. The new stabilization algorithm is described in the literature [20-22], which could be used to generate the entire set of stabilizing PID controllers.

Consider the feedback system with PID controller, which is shown in Fig. 2, the plant transfer function \( G(s) \) is expressed as:

\[
G(s) = \frac{N(s)}{D(s)}
\]  

where \( N(s) \) and \( D(s) \) are polynomials in the Laplace variable \( s \). The closed-loop characteristic polynomial is:

\[
\delta(s, k_p, k_i, k_d) = sD(s) + (k_i + k_ds^2)N(s) + k_psN(s)
\]  

The problem of the PID controller design stabilizing the given plant has been converted to determining the values of \( k_p, k_i, \) and \( k_d \) for which the closed-loop characteristic polynomial is Hurwitz, which means it has all its roots in the open left half plane (LHP).

The basis idea of the root counting theory is separating the parameters. Define

\[
N^*(s) = N(-s) = N_o(s^2) - sN_o(s^2)
\]  

Then define,

\[
v(s) := \delta(s, k_p, k_i, k_d)N^*(s)
\]  

\[
= \left[ s^2 \left( N_o(s^2)D_o(s^2) - D_o(s^2)N_o(s^2) \right) + \left( k_i + k_ds^2 \right) \left( N_e(s^2)N_o(s^2) - s^2N_o(s^2)N_o(s^2) \right) \right]
\]

\[
+ s \left[ D_o(s^2)N_o(s^2) - s^2D_o(s^2)N_o(s^2) \right] + k_p \left( N_o(s^2)N_e(s^2) - s^2N_o(s^2)N_o(s^2) \right)
\]  

The stability of \( \delta(s, k_p, k_i, k_d) \) is determined by whether \( v(s) \) has exactly the same number of closed right half plane (RHP) zeros as \( N^*(s) \). By creating the test polynomial \( v(s) \), the parameter separation has been achieved, where \( \delta(s, k_p, k_i, k_d) \) has all three parameters appearing in both the even and odd parts, comparing with \( v(s) \) that only has \( k_p \) appearing in the odd part, and \( k_i, k_d \) appearing in the even part. Let \( s = j\omega \):

\[
\delta(j\omega, k_p, k_i, k_d)N^*(j\omega) = p(\omega, k_p, k_i) + jq(\omega, k_p)
\]  

where \( p(\omega, k_p, k_i) = p_1(\omega) + (k_i - k_d\omega^2)p_2(\omega), \quad q(\omega, k_p) = q_1(\omega) + k_pq_2(\omega), \quad p_1(\omega) = -\omega^2(N_o(-\omega^2)D_o(-\omega^2) - D_o(-\omega^2)N_o(-\omega^2)) \]

\[
p_2(\omega) = N_o(-\omega^2)N_e(-\omega^2) + \omega^2N_o(-\omega^2)N_o(-\omega^2) \]

\[
q_1(\omega) = \omega D_o(-\omega^2)N_o(-\omega^2) + \omega^2D_o(-\omega^2)N_o(-\omega^2) \quad q_2(\omega) = \omega N_o(-\omega^2)N_e(-\omega^2) + \omega^2N_o(-\omega^2)N_o(-\omega^2)
\]

The subscript of \( o \) and \( e \) are the components of odd and even part respectively.

Let \( l(N(s)) \) and \( r(N(s)) \) denote the number of roots of \( N(s) \) in the open LHP and RHP respectively; let \( n \) and \( m \) be the degrees of \( \delta(s, k_p, k_i, k_d) \) and \( N(s) \) respectively.

\[
sgn : R \rightarrow \{-1, 0, 1\} \text{ is the standard signum function. Since } v(s) \text{ must have exactly the same number of RHP roots as } N^*(s), \text{ the range of } k_p \text{ will be determined by satisfying the necessary condition that } q(\omega, k_p) \text{ has at least } \beta \text{ real, nonnegative, distinct roots of odd multiplicity, where}
\]

\[
\beta = \begin{cases} 
\frac{n - l(N(s)) - r(N(s))}{2} & \text{for } m + n \text{ even} \\
\frac{n - l(N(s)) - r(N(s)) + 1}{2} & \text{for } m + n \text{ odd}
\end{cases}
\]

By using this root counting theory, the stability condition reduces to:
\[
\begin{align*}
 n - (l(N(s)) - r(N(s))) &= \left\{ \begin{array}{ll}
 i_n - 2i_1 + 2i_2 + \cdots + (-1)^{n-1}2i_{n-1} \\
 + (-1)^{n}i_n & \text{for } m+n \text{ even} \\
 i_n - 2i_1 + 2i_2 + \cdots + (-1)^{n-1}2i_{n-1} \\
 - (-1)^{n}i_n & \text{for } m+n \text{ odd}
\end{array} \right.
\end{align*}
\]

In conclusion, the value of \( k_p \) will be fixed by choosing from the range of \( k_p \), which is determined by roots counting of \( q(\omega, k_p) \). Then for each fixed value of \( k_p \), determining the set of \( \{ k_r, k_d \} \) values that simultaneously satisfy the following string of linear inequalities:

\[
[p_i(\omega) + (k_r - k_d \omega^2)p_2(\omega)] q_i > 0, \forall t = 0, 1, 2, \ldots
\]

By sweeping \( k_p \) over the appropriate ranges, the entire set of stabilizing the plant by the designed PID controller is determined. The detailed proof and formulas of root counting theory can be found in the literature [21].

IV. IMPLEMENTATION OF STATCOM CONTROL SYSTEM

A. Implementation Approach for PID Design

The PID design approach based on the root counting is aimed at single-input single-output. However, STATCOM control system requires the multivariable control approach because of the MIMO characteristic. As mentioned in Section II, Park transform is used to decouple the multivariable system. Park transform is a very powerful tool in power system analysis. It can convert the three phase voltages and currents into orthogonal components in a synchronous rotating frame. Those orthogonal components are fixed to the rotor and are only related with active and reactive power respectively, which is shown in equation (13):

\[
\begin{bmatrix}
    i_d \\
    i_q \\
    0
\end{bmatrix}
= P
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix},
\]

where \( i_a, i_b, \text{ and } i_c \) are the currents for three AC phase respectively;

\( i_d \) and \( i_q \) are active and reactive components in the rotating frame;

\( P \) is the Park transform matrix:

\[
P = \frac{2}{3}
\begin{bmatrix}
    \cos(\omega t) & \cos(\omega t - \frac{\pi}{3}) & \cos(\omega t + \frac{\pi}{3}) \\
    -\sin(\omega t) & -\sin(\omega t - \frac{\pi}{3}) & -\sin(\omega t + \frac{\pi}{3}) \\
    \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

Thus the control design problem will be greatly simplified using this decoupling processing. Literature presents the detail mathematical model for decoupling [15]. The new design approach presented in this paper can be used for each decoupled component.

Root counting theory provides a feasible way to get the entire set of controls for stabilizing the system. This method highly depends on the computer aided design. With the mature development of computer techniques, it will be much easy to implement the design. Once the entire stabilizing set has been determined, the extra requirements such as guaranteed rising time, settling time, overshoot, gain and phase margins can be easily obtained.

B. Performance Requirements for STATCOM Controller

STATCOM is an important part of the Flexible AC Transmission System (FACTS) solution, which is used to improve power system stability [26, 27]. It is widely used for voltage regulation by injecting or absorbing system reactive power. Thus the STATCOM controller is required to maintain the output voltage or current tracking via regulation of input components. The desired dynamic performances for STATCOM controller design include less oscillation, smaller overshoot, and shorter settling time. The control strategy focuses on performance requirements for each of the decoupled components.

In this paper, the performance requirements are considered in proposed PID controller design method, which includes rising time, settling time, overshoot, gain and phase margins. After the entire set for the stabilizing system based on root counting and signature theory is obtained, the quantity values of each performance specifications will be calculated point by point in the entire set. The comparison criterion is created according to the performance requirements of PID controller design. All the points whose performance meets the criterion will be recorded. Thus by sweeping point by point, the subset within which the points satisfy the performance requirements will be obtained.

V. CASE STUDY SIMULATION

In order to verify the proposed approach, a Matlab model for simulating the system shown in Fig. 2 is built to test this design method. The chosen system parameters are listed below, which came from [14]:

- Line resistance \( R = 2\mu \Omega \)
- Line inductance \( L = 400\mu H \)
- Frequency \( f = 60Hz \)

Therefore the corresponding coefficient matrices for the state space transfer function are:

\[
A = \begin{bmatrix}
    -5 & 376.99 \\
    -376.99 & -5
\end{bmatrix}, \quad B = \begin{bmatrix}
    2500 & 0 \\
    0 & 2500
\end{bmatrix}, \quad C = \begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix}
\]

The proposed PID controller design approach is tested by the Matlab model for STATCOM. Root counting theory is used for each of the decoupled components. We take the controller for the variable $i_d$ as the example. Fig. 3 shows the entire stabilizing set of PID controller. Any point within the obtained set refers to one PID controller, whose coefficients values are $(k_p, k_i, k_d)$. This PID controller can stabilize the studied STATCOM. Fig. 4 shows an example for STATCOM unit step response for stabilizing and un-stabilizing PID controller which come from the point of inside and outside the stabilizing set respectively.

Fig. 3. The Stabilizing set of $(k_p, k_i, k_d)$ values

![Graph showing the stabilizing set of PID coefficients](image)

Fig. 4. The unit step response for stabilizing and un-stabilizing PID Controller

![Graph showing unit step response](image)

Stabilizing the plant is the necessary requirement for PID controller design. The performance attainment problems are also very important, especially in industry application. As we mentioned before, one the entire stabilizing set of the coefficients of PID controller has been obtained, the performance test can be implement for each of the point. The performance specification can be added to the PID controller design. Table I shows the six sets of PID controller coefficients for the case study.

The unit step response for each of the PID controllers is shown in Fig. 5. They exhibit different performance although all of them can stabilize the studied plant of STATCOM. For example, in some cases we hope to get the PID controller with overshoot less than 0.05%. Fig. 6 shows the results of the overshoot for the set of six PID controllers. Under this requirement, PID 2, 3, 4 and 5 are satisfied. PID 3 has the best overshoot performance among these PID controllers.

<table>
<thead>
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<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
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<tbody>
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<td>496.164</td>
<td>44.3262</td>
<td>100.806</td>
</tr>
<tr>
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<td>26.5957</td>
<td>52.4294</td>
</tr>
<tr>
<td>273.657</td>
<td>79.7872</td>
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</tr>
<tr>
<td>273.657</td>
<td>186.170</td>
<td>52.4294</td>
</tr>
<tr>
<td>7.673</td>
<td>8.8625</td>
<td>100.806</td>
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</tbody>
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TABLE I

PID CONTROLLER COEFFICIENTS

<table>
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<th>$K_i$</th>
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</tr>
<tr>
<td>6</td>
<td>186.170</td>
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</tr>
</tbody>
</table>

Fig. 5. Unit step response for each PID controller of $i_d$

![Graph showing unit step response for each PID controller](image)

Fig. 6. The performance of overshooting of each PID controller

VI. CONCLUSION

STATCOM has become one of the most important equipments for the reactive power compensation. Fast response controller with well-designed coefficients for the STATCOM control will improve the stability of power system operation and decrease the possibility of blackouts happening, which will be beneficial for preventing cascading events.

This paper presents a new approach for PID controller design of STATCOM, which is based on root counting and
signature theory. This approach can locate the complete set of stabilizing PID controllers in the PID coefficients gain space, $(k_p, k_i, k_d)$, and any point within this set exhibits an ability to stabilize the studied STATCOM system. The performance attainment problems will be easily solved after the entire stabilizing set is found. The desired coefficient set of PID controller for satisfying the performance requirements of STATCOM could be identified by sweeping the part of entire set, which includes overshoot, rising time, settling time, gain and phase margins. Case study has been implemented based on the Matlab simulation. The test results verified the feasibility of the proposed approach.

VII. ACKNOWLEDGMENT

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VIII. REFERENCES


IX. BIOGRAPHIES

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