A Novel Measure of Component Importance Considering Cost for All-Digital Protection Systems

Peichao Zhang, *Member, IEEE*, Chao Huo and Mladen Kezunovic, *Fellow, IEEE*

***Abstract*— Component importance analysis plays an important role in system reliability theory. This paper proposes a novel measure of component importance for all-digital protection systems, which considers both the manufacturing cost and the failure cost. First, the function of manufacturing cost versus component failure rate is proposed, and the function of total cost versus system reliability for all-digital protection systems is derived. Second, the measure of importance considering cost with respect to component as well as component type is defined, and related calculation equations are derived. Afterwards, the component importance for a typical all-digital protection system is analyzed and discussed, which demonstrates the application value of the new measure of component importance. A conclusion about the benefits of the application principle and method of the new measure is given at the end.**

***Index Terms*— Component importance, three parameters model, reliability, all-digital protection system, cost**

Acronyms[1]

TS	time source
MU	merging unit
PR	protective relay
SW	Ethernet switch
EM	Ethernet communication media
RBD	reliability block diagram

Notations

λ_i	failure rate of component i
$R_i(t), p_i(t)$	reliability of component i
C	system total cost
C_i	manufacturing cost of component i
C_{0i}	initial manufacturing cost of component i
$R_{sys}(t)$	system reliability
$Q_{sys}(t)$	system unreliability
$W(0,t)$	the expected number of failures during [0,t)
C_f	cost of system failure
$I^B(i\|t)$	Birnbaum's measure of importance of component i
$I_\lambda^C(i\|t)$	measure of importance considering cost of component i
$I_\lambda^C(\beta\|t)$	measure of importance considering cost with respect to type β

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[1]The singular and plural of an acronym are always spelled the same.

I. Introduction

Recent development of non-conventional instrument transformers and high-speed Ethernet technology permits implementation of an all-digital protection system [1]. Compared with the conventional protection system, an all-digital protection system typically comprises more electronic devices, e.g. , merging units, Ethernet switches and time synchronization sources. As a result, the reliability of an all-digital protection system should be examined from not only the system level but also the component level. With respect to the component level, analysis of the importance of various components of the all-digital protection system should be a key part of the system reliability quantification process. Two measures of the importance, namely the Birnbaum's importance and the criticality importance, have been investigated and suggested as reliability indices for critical components of all-digital protection systems [2].

The classical component importance analysis considers the reliability only and pays little attention to economic factors [3], [4], [5]. In practice, it is unrealistic to evaluate the importance of components and to improve the system reliability without considering the cost [6]. This paper aims at proposing a new measure of component importance for all-digital protection systems, which considers both the manufacturing and failure cost. The paper is organized as follows. Section II introduces a typical system architecture of the all-digital protection system. Section III derives the model of cost versus reliability and proposes the novel measure of component importance considering cost. Section IV presents the simulation results. Section V concludes the paper.

II. System Architecture and Assumptions

Six alternative architectures of all-digital protection systems have been defined and discussed in detail in [2]. In this paper we select one of them, as shown in Fig. 1, for demonstration purpose. A reliability block diagram (RBD) is a success-oriented network describing the function of the system [7], [8]. It shows the logical connections of components needed to fulfill a specified system function. Fig. 2 shows the related RBD of the system shown in Fig. 1. Based on the RBD, the minimal path set and the connection matrix technology are adopted to derive the system reliability function $R_{sys}(t)$, which have been demonstrated in detail in [2].

The paper makes the following assumptions.

1) The failure rate of each component is a constant. The reliability of component i is thus:

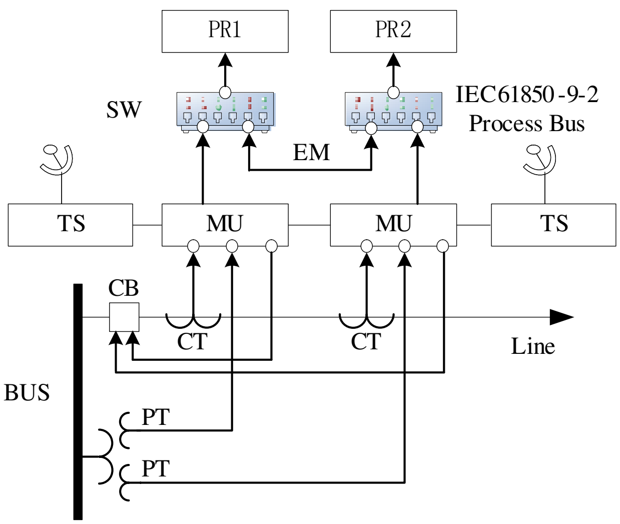


Fig. 1. A typical architecture of the all-digital protection system.

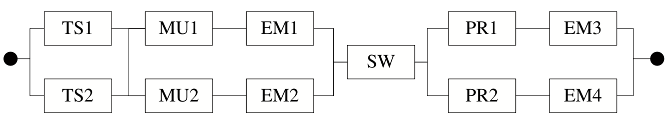


Fig. 2. Reliability Block Diagram of an all-digital protection system.

$$R_i(t) = p_i(t) = e^{-\lambda_i t}. \tag{1}$$

where λ_i is the failure rate of component i.

2) Reliability of protective relaying is a compromise between security and dependability [9]. The dependability, as well as the security issue has been discussed in [2]. For simplicity, we study only the dependability issue in this paper.
3) The life time of the protection system consists of repeated cycles of failure and repair. Since this paper aims at assessing the relative importance among the components, we consider only the failure process and properties of the all-digital protection system. Note that the system is normally assumed to be non-repairable while doing the component importance analysis.

As noted above, an actual protection system is repairable and will be inspected periodically. Under the assumptions that the inspection always detects failures and repair always restores the protection to "as good as new" status, the system reliability over time is shown in Fig. 3 [9]. In Fig. 3, T is the inspection period and τ is the time occupied in every inspection. We limit the studied mission time in one inspection cycle so that the system can be approximately considered as a non-repairable one in the given period.

Suppose T is 1800 hours and the expected life time of an all-digital protection system is 20 years, the total number of inspections in the life time is thus: $N = 20 \times 24 \times 365 \div 1800 = 97.33$.

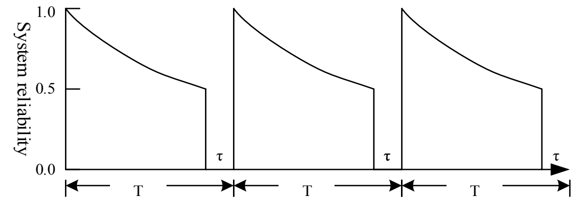


Fig. 3. System reliability of the protection system with periodic inspection.

III. Component importance considering cost

A. Model of cost versus reliability of components

Several classical cost-reliability models (e.g., Lagrange model and power model [10]) have been proposed in the past. Lagrange model depends on the condition that the logarithm of component unreliability is proportional to cost, which may not always be true in reality. Power model includes two constants to be calculated, both of which have little relationship with reliability and are hard to obtain. So it is difficult to use these models in practice.

The "three parameters model" was proposed in [11], which is defined as

$$C_i = EXP[(1 - f_i)\frac{R_i - R_{imin}}{R_{imax} - R_i}]. \tag{2}$$

This is an exponential function of manufacturing cost with respect to reliability which contains three parameters, namely, f_i, R_{imin} and R_{imax}. The first parameter, $f_i \in [0, 1]$, is the feasibility of increasing a component's reliability. Some methods used for obtaining the feasibility are summarized in [12]. The second parameter, R_{imin}, is the initial (current) reliability value of the ith component at the specific time. The third parameter, R_{imax}, is the maximum achievable reliability of the ith component.

Two shortcomings still exist in the "three parameters model":

1) It assumes the manufacturing cost of the component equals 1, when $R_i = R_{imin}$.
2) This model expresses the cost in terms of the parameter R_i, which is not a constant but a function of time t. It is not convenient when using this model since typically manufacturing cost is not a function of the mission time.

To solve the shortcomings, we propose a new cost-reliability model. Let C_{0i} denote the initial (current) manufacturing cost, and substitute (1) into (2), we get

$$C_i' = C_{0i} \cdot EXP[(1 - f_i)\frac{e^{-\lambda_i t} - e^{-\lambda_{imax} t}}{e^{-\lambda_{imin} t} - e^{-\lambda_i t}}]. \tag{3}$$

The interested mission time t in this paper ranges from 0 hour to $10,000$ hours and the component failure rate λ_i ranges from 0.01 year^{-1} to 0.1 year^{-1}. Since $\lambda t \leq 0.11$, we have

$$e^{-\lambda t} \approx 1 - \lambda t. \tag{4}$$

By substituting (4) into (3), we derive a new cost function which expresses the manufacturing cost in terms of failure rate,

$$C_i^{\lambda} = C_{0i} \cdot EXP[(1-f_i)\frac{\lambda_{imax}-\lambda_i}{\lambda_i-\lambda_{imin}}]. \quad (5)$$

The new model has the following characteristics:

- Compared with the old model expressed by (2), the new model represents the cost in terms of the constant failure rate rather than the time-varying R_i, which makes the model more convenient for everyday use.
- C_i^{λ} approaches infinity while the component failure rate λ_i approaches λ_{imin}, whereas it will be very low while λ_i is very high.
- C_i^{λ} is monotonically decreasing while λ_i is increasing. Similarly, ΔC_i^{λ} is monotonically decreasing while $\Delta\lambda_i$ is increasing.

B. Model of total cost versus reliability

In this paper, the total cost comprises of two parts:

- the manufacturing cost of the protection system which is assumed to be the sum of individual component costs;
- the failure cost which is the loss caused by the operational failure of the protection system, e.g., a permanent loss of generator life [9].

The maintenance cost (e.g., the inspection and repair cost) of the protection system is ignored since only the failure process of the system is considered. For a non-repairable system, the expected number of failures during $[0,t)$ can be calculated by:

$$W(0,t) = \int_0^t w(t)dt = 1 - R_{sys}(t), \quad (6)$$

where $w(t)$ is the frequency of failures [9].

Let C_j denote the expected loss caused by an operational failure of the protection system, the expected loss of failure during $[0,t)$ can thus be calculated as follows:

$$C_f = C_j \cdot W(0,t) = C_j(1 - R_{sys}(t)). \quad (7)$$

Let C_c denote the manufacturing cost of the protection system. Using (5) and (7), the total cost considering both the manufacturing cost and the cost of failure is thus:

$$\begin{aligned} C =& C_c/N + C_f \\ =& \sum_{i=1}^{n} C_{0i} \cdot EXP[(1-f_i)\frac{\lambda_{imax}-\lambda_i}{\lambda_i-\lambda_{imin}}]/N \\ &+ C_j(1-R_{sys}(t)), \end{aligned} \quad (8)$$

where N represents the number of inspections during the life time of the protection system and thus C_c/N represents the average manufacturing cost apportioned in one inspection period. Note again that we limit the mission time in one inspection period so that the system can be approximately considered as a non-repairable one.

C. Component importance considering cost

Several classical measures of importance have been defined to assess component importance [13], [14], [15]. Among them, the Birnbaum's importance [13] is the most popular one, which is defined as

$$\begin{aligned} I^B(i|t) &= h(1_i, R_{sys}(t)) - h(0_i, R_{sys}(t)) \\ &= \frac{\partial R_{sys}(t)}{\partial p_i(t)}, \end{aligned} \quad (9)$$

where $h(1_i, R_{sys}(t))$ denotes the conditional probability that the system is functioning when it is known that component i is functioning at time t, and $h(0_i, R_{sys}(t))$ denotes the conditional probability that the system is functioning when component i is in a failed state at time t.

In this paper, we propose a novel measure of component importance, which is defined as the partial derivative of the total cost with respect to the partial derivative of th failure rate of the component. The expression is

$$I_{\lambda}^{C}(i|t) = \frac{\partial C}{\partial \lambda_i}, \quad (10)$$

where $i = 1, 2, \ldots, n$.

Substituting (8) in (10) and then using (5), we have

$$\begin{aligned} I_{\lambda}^{C}(i|t) =& \frac{\partial C}{\partial \lambda_i} = \frac{\partial C}{\partial p_i} \cdot \frac{\partial p_i}{\partial \lambda_i} \\ =& \frac{1}{N} \cdot \frac{\partial C_i^{\lambda}}{\partial \lambda_i} + C_j \cdot \frac{\partial R_{sys}(t)}{\partial p_i(t)} t p_i(t) \\ =& C_j \cdot I^B(i|t) t e^{-\lambda_i t} - C_i^{\lambda} \frac{(1-f_i)(\lambda_{imax}-\lambda_{imin})}{N(\lambda_i-\lambda_{imin})^2}. \end{aligned} \quad (11)$$

As seen in (11), the importance of component i when considering cost depends on four factors: (1) the location of the component in the system; (2) the reliability of the component in question; (3) the cost of improving the reliability of the component; (4) the loss caused by the operational failures of the protection system. The former two factors are measured by classical component importance, whereas the latter two factors are measured by the novel model of component importance. Accordingly, the classical component importance measures the sensitivity of a component in terms of reliability only and the novel component importance is more comprehensive by taking both reliability and economic factors into consideration.

D. Importance considering cost with respect to component type

In a practical all-digital system, there may be multiple identical components (e.g., Ethernet switches) in different positions in the system. It is useful to identify the component type as a unit which contributes the most to system reliability [16]. Consider a system with n independent components, in which there are m identical components of type β. Let $p_{i1}(t) = p_{i2}(t) = \cdots = p_{im}(t) = p_{\beta}(t)$ denote the reliability of these m identical components, and $\lambda_{i1} = \lambda_{i2} = \cdots = \lambda_{im} = \lambda_{\beta}$ denote the failure rate of these m identical components. Then

TABLE I
RELIABILITY DATA OF THE COMPONENTS

Component	C_{0i} (k $dollars$)	λ_i ($1/yr$)	λ_{imin} ($1/yr$)	λ_{imax} ($1/yr$)	f_i
TS	1.25	0.02	0.008	0.03	0.8
MU	5	0.02	0.008	0.03	0.9
SW	1.25	0.02	0.008	0.03	0.8
PR	10	0.02	0.008	0.03	0.9
EM	0.25	0.01	0.005	0.02	0.7

the importance considering cost with respect to component type is:

$$\begin{aligned} I_\lambda^C(\beta|t) &= \frac{\partial C}{\partial \lambda_\beta} \\ &= \sum_{i=1}^{n} \frac{\partial C}{\partial \lambda_i(t)} \cdot \frac{\partial \lambda_i(t)}{\partial \lambda_\beta(t)} \\ &= \sum_{l=1}^{m} \frac{\partial C}{\partial \lambda_{il}(t)} \cdot \frac{\partial \lambda_{il}(t)}{\partial \lambda_\beta(t)} \\ &= \sum_{l=1}^{m} I_\lambda^C(il|t). \end{aligned} \quad (12)$$

As shown in (12), $I_\lambda^C(\beta|t)$ is simply the sum of the importance of all the components of the same type.

IV. CASE STUDY

The system shown in Fig.1 is used in the case study. The reliability data listed in Table. I is used in the study unless otherwise noted. Besides, suppose $N = 97.33$, $C_j = 600,000$ dollars per operational failure, and the interested mission time at which the system is observed is $t = 1800$ hours.

A. Relationship between C and λ_i

Taking component EM as an example, the diagram of $C \sim \lambda_i$ is shown in Fig. 4. It can be seen that, while λ_i is far away from λ_{imin}, the manufacturing cost changes slightly when λ_i changes. While λ_i approaches λ_{imax}, the arising of the failure rate will result in the arising of the total cost, even though the manufacturing cost will decrease in this situation. On the contrary, while λ_i approaches λ_{imin}, a slight change of λ_i will result in very remarkable change of the total cost, since the manufacturing cost will increase rapidly. While λ_i reduces to some extent, the total cost almost depends on the manufacturing cost only. Figure. 4 also shows the optimal value of λ_i which corresponds to the minimal total cost can be determined by locating the lower-most point in the curve of total cost.

B. Component importance considering cost ($\partial C/\partial \lambda_i$)

The simulation result of the component importance considering cost is shown in Fig. 5. For purpose of comparison, the Birnbaum's importance is shown in Fig. 6. The results and orders of the two types of component importance at $t = 1800$ hours are shown in Table II.

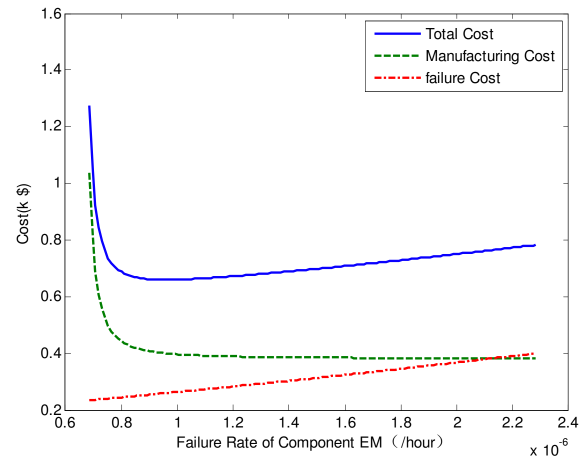


Fig. 4. Simulation result of cost vs. failure rate.

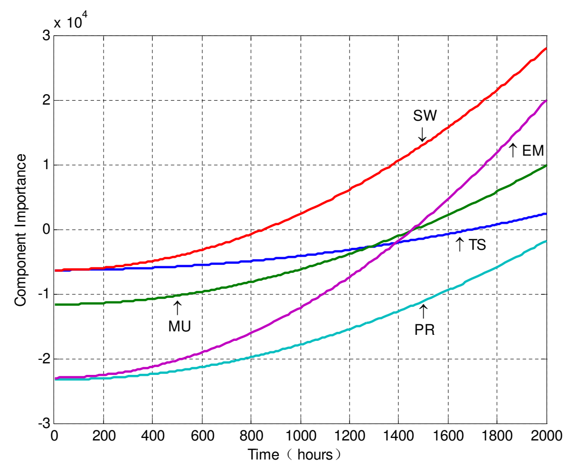


Fig. 5. Component importance with respect to type considering cost.

Based on the experimental results, some conclusions are derived as follows:

1) As seen in Table II, the two types of measures may lead to different rankings. The Birnbaum's measure may give the same ranking of the components (e.g., the merging unit and the protective relay) in some cases, whereas the new measure will always result in a more dynamic and deterministic rankings of the components.
2) The Birnbaum's measure is always positive since the protection system is a coherent system, which means all the components are relevant and the system reliability is bound to be improved when improving the reliability of the components. On the contrary, the new measure of importance may lead to negative values, such as the importance of PR shown in Table II, which is discussed further as follows:
   - If the importance of component i considering cost is positive, meaning the total cost will decrease when the component failure rate λ_i decreases, we should consider to increase the reliability (by decreasing

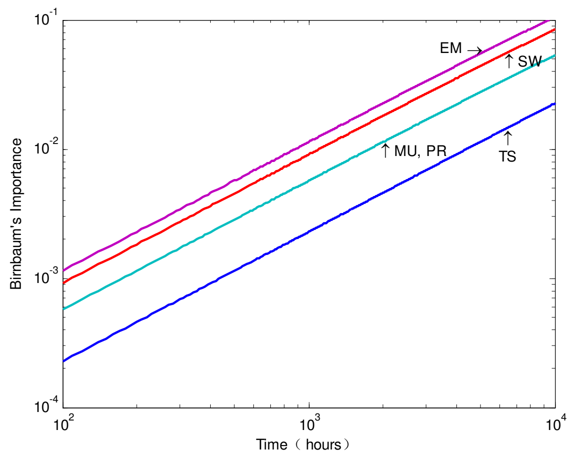
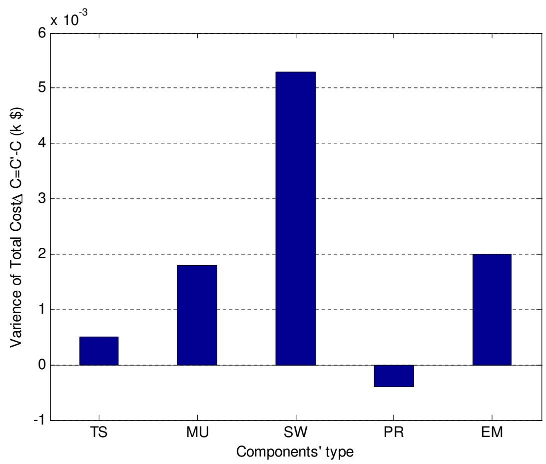
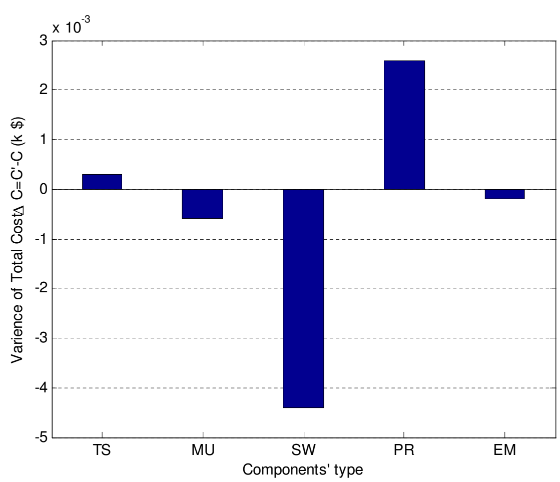


Fig. 6. Classical component importance.



(a) Each component's λ_i increases 10% seperately



(b) Each component's λ_i reduces 10% seperately

Fig. 7. Impact of the variation of failure rate of each component on the total cost (t=1800 hours).

TABLE II
RESULTS AND ORDERS OF THE TWO TYPES OF COMPONENT IMPORTANCE (T=1800 HOURS)

	TS	MU	SW	PR	EM
Classical	0.0041	0.0101	0.0162	0.0101	0.0202
importance	Order: EM > SW > (MU = PR) > TS				
the new	9.15e+2	7.25e+3	2.69e+4	-7.30e+3	1.49e+4
importance	Order: SW > EM > MU > TS > PR				

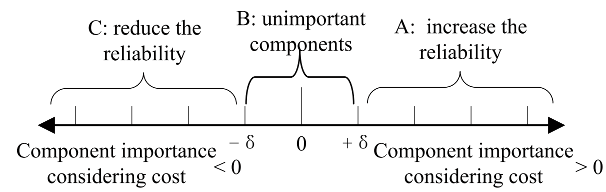


Fig. 8. Application principles of component importance considering cost.

the failure rate) of the component to reduce the total cost.

- If the importance of component i considering cost is negative (e.g., the protective relay), meaning that the total cost will decrease when λ_i increases, we should consider to lower the reliability (by increasing the failure rate) of the component to reduce the total cost under an assumption that the system reliability meets the requirement.
- If the reliability optimization can only be made to component i, the optimal λ_i corresponding to the minimal total cost can be determined by finding the solution of $I_\lambda^C(i|t) = \partial C/\partial\lambda_i = 0$. Taking EM as an example, the optimal λ_i is 0.0085 year^{-1}.

To further demonstrate the above discussions, we decrease and increase each component's failure rate independently by 10%, then calculate the variances between the new total cost and the old one. From the results plotted in Fig. 7, it can be seen that the impact of the variation of the failure rate of each component on the total cost is approximately consistent with the component's importance shown in Table II. It should be noted that, the reason they are not completely consistent is just $\Delta C/\Delta\lambda_i \neq \partial C/\partial\lambda_i$ when $\Delta\lambda_i$ equals 10%.

We conclude the discussions by summarizing the application principle and method of the novel measure as follows:

1) Ranking the components by their importance and grouping them into three sets, namely A, B and C as shown in Fig. 8. $A = \{i|I_\lambda^C(i|t) > \delta\}$, $B = \{i|I_\lambda^C(i|t) \in [-\delta, \delta]\}$, $C = \{i|I_\lambda^C(i|t) < -\delta\}$, where i represents the component in question, t represents the interested mission time and δ is a constant used to group the components.
2) The components in set A are considered as important in the sense of the need to improve their reliability. The component with the highest importance value should be considered first.
3) The components in set B are considered as unimportant ones and there is no need to change their reliability as they have little impact on the total cost.

4) The components in set C are also considered as unimportant since they are not the candidates whose reliability should be improved. Unlike the components in set B, the reliability of which should be neither increased nor decreased, the reliability of components in set C can be decreased to reduce the total cost under an assumption that the system reliability meets the requirement. The component with the lowest importance value should be considered first.

V. Conclusion

As compared with the classical measure of component importance, the novel measure proposed in this paper considers not only the location of the component in the system and the reliability of the component, but also the manufacturing cost of the components and the loss caused by the system operational failure. The new measure is more informative and comprehensive to guide the designers when trying to optimize system reliability. Besides, the novel measure is more dynamic and responsive in the sense of being able to provide importance ranking for the importance of the components included in the all-digital protection systems.

The methodology proposed in this paper is easy to implement using software and suitable to analyze for an all-digital protection system with arbitrary architectures. If being appropriately applied, it will also be suitable for analysis of the component importance of digital substations.

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